

Practice Questions

(3)

1. Find the following values:

$6 \bmod 13 = 6$ $\frac{6}{13} = 0 \text{ R } 6$

$42 \bmod 7 = 0$ $42/7 = 6 \text{ R } 0$

$30 \bmod 4 = 2$ $30/4 = 7 \text{ R } 2$

2. Decide if the following are true or false:

$12 \equiv 2 \bmod 10$ is T $12 - 2 = 10$

$53 \equiv 2 \bmod 13$ is F $53 - 2 = 51$

and 51 is not evenly divisible by 13

(d) Single digit error in 3rd position?

$3a_1 + a_2 + 9a_3 \equiv 3a_1 + a_2 + 9b_3 \pmod{10}$

$9(a_3 - b_3) \equiv 0 \pmod{10}$

will catch all single digit errors

b/c $(a_3 - b_3) \times 9$ will not give a

multiple of 10

3. A code is given by $a_1 a_2 a_3 a_4$ where a_4 is the check digit. Given that $a_4 = 3a_1 + a_2 + 7a_3 \pmod{10}$,

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(a) Find the check digit for the code 123.

$a_4 = 3(1) + 2 + 7(3) = 26 \equiv 6 \pmod{10}$

$a_4 = 6$

(b) Determine the value of x in the code $34x7$ given that the check digit is valid.

$a_4 = 9 \pmod{10} = 9, 17, 27, \text{etc}$
 $= 3(3) + 4 + 7(x) = 13 + 7x$

- $13 + 7x = 17 \rightarrow \text{NO}, 13 + 7x = 27 \rightarrow x = 2$
- $13 + 7x = 37 \text{ NO}, 13 + 7x = 47 \text{ NO}, 13 + 7x = 57 \text{ NO}$
- $13 + 7x = 67 \text{ NO}, 13 + 7x = 77 \text{ NO}$
- NOTE $13 + 7x = 96$ is max

(c) Will this code find the error of transposing the first two digits?

$3a_1 + a_2 + 9a_3 \not\equiv 3a_2 + a_1 + 9a_3 \pmod{10}$

$3a_1 + a_2 + 9a_3 - 3a_2 - a_1 - 9a_3 \not\equiv 0 \pmod{10}$

$2a_1 - 2a_2 \not\equiv 0 \pmod{10}$

$2(a_1 - a_2) \not\equiv 0 \pmod{10}$

if a_1 and a_2 differ by 5, won't catch transposing. yes in other cases

(d)

4. A ZIP+4 code has a check digit in the 10th position that is found by summing the first 9 digits and the check digit will bring the sum of all the digits to a multiple of 10. What is the check digit for a ZIP+4 of 77845-8738?

$7+7+8+4+5+8+7+3+8 = 57$
 Check digit is 3 b/c $57+3=60$

5. An ID number has 5 digits where the first two digits are the last two digits of the person's zip code, the 3rd digit is the person's birth month mod 10 and the last two digits are the person's birth day. What is the ID number of a person who lives in the zip code 77840 and is born on November 28, 1983?

$$\begin{array}{r} 4 \\ \hline 77840 \end{array} \quad \begin{array}{r} 1 \\ \hline 28 \end{array}$$

 Nov $\rightarrow 11 \text{ mod } 10 = 1$

6. Convert the binary number 10101 to a decimal number.

$1 \times 16 + 0 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 = 21$
 What is 97 expressed as a binary number?
 $97 = 1 \times 64 + 1 \times 32 + 0 \times 16 + 0 \times 8 + 0 \times 4 + 0 \times 2 + 1 \times 1$
 1100001

7. What is the code word for the message 11001 if the code word is the message appended with two check digits found using the parity-check sums $a_1 + 2a_2 + a_3$ and $3a_1 + a_3 + a_5$.

$a_1 + 2a_2 + a_3 = 1 + 2(1) + 0 = 3 \text{ (odd)} \rightarrow 1$
 $3a_1 + a_3 + a_5 = 3(1) + 0 + 1 = 4 \text{ (even)} \rightarrow 0$
 1100110

8. A code $C = \{11111111, 11110000, 00001111, 00000000\}$ can send 4 different messages.

(a) What is the weight of this code? 4
 (b) How many errors can this code detect? $4-1=3$

(c) Use nearest neighbor decoding to decode, if possible, the message 11100111.

11111111 } dist of 2 \leftarrow this is it
 11100111
 11110000 } dist of 4
 11100111
 00001111 } dist of 4
 11100111
 00000000 } dist of 6
 11100111

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9. Data was compressed using the encoding
e → 0, s → 10, t → 110 and a → 111.

Decode the message



10. Decode the information that used delta encoding:

29 9 -4 0 -3 -3 10. → 12char

29 38 34 34 31 28 38
14char - saves (8)

Encode the values 107 102 103 105 100 using delta encoding.

107 -5 1 2 -5

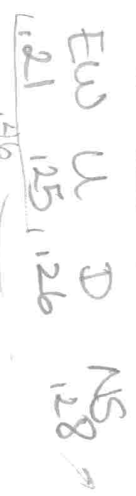
15 char down to 9 → saves (6)

In each case, how much compression was there?

(8)

11. Use Huffman encoding to create a binary code for the directions that occur with the following frequencies:

East West North South Up Down
 0.10 0.11 0.13 0.15 0.25 0.26



E = 0000
 W = 0001
 U = 01
 D = 10
 N = 110
 S = 111

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P

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12. Use the Caesar cipher with a shift of 16 to encrypt the message

D I P L O M A
T Y F B E C Q

13. Use a decimation cipher with key 3 to encrypt the message

N E W Y E A R.
position 13 4 22 24 4 0 17
x3 39 12 66 72 12 0 51
N M O U M A Z

14. Use the Vigenere cipher with the key word HANK to encrypt the message

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H	O	M	E	R	U	N.	
7	14	12	4	17	20	13	position
H	A	N	K	H	A	N	KEY position
7	0	13	10	7	0	13	
14	14	25	14	24	20	26	sum
14	14	25	14	24	20	0	mod 26
Q	Q	Z	Q	Y	U	A	letter

15. Given that TROUT was used as the key word to encrypt ZFBY YBJVCGZ with the Vigenere cipher, decrypt the message.

Z	F	B	Y	Y	B	T	V	C	G	Z	
25	5	1	24	24	19	21	2	6	25		position
T	R	O	U	T	T	R	O	U	T		KEY position
19	17	14	20	19	19	17	14	20	19		position
6	-12	-13	4	5	-18	-8	9	-18	-8	6	substrait
6	14	13	4	5	8	18	9	8	13	6	mod 26
G	O	N	E	F	I	S	H	I	N	G	letter

16. Add the binary numbers 11111 and 10101.

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11111
10101
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01010

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