MATH 142 Business Calculus

Fall 2019, WEEK 10

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Week 10 Section 5.8, 6.1 Implicit Differentiation and Related Rates, Antiderivative

Section 5.8 Implicit Differentiation and Related Rates

Why we differentiate Implicitly?

Suppose $y = f(x)$. In this case, we say y is an explicit function of x and we can therefore differentiate as usual: $\frac{dy}{dx} = f'(x)$. In this section, we investigate how to differentiate if y cannot be written as an explicit function of x, that is y is implicitly defined as a function of x. Fortunately, we don't need to solve an equation for y in terms of x in order to find the derivative of y. Instead we can use the method of **implicit differentiation.** This consists of differentiating both sides of the equation with respect to x and then solving the resulting equation for y'.

Ex.1) Find $\frac{dy}{dx}$ for the following equations.

(a) $x^2 + y^2 = 4$

(b) $y^3 = 2x^2 + y^4$

(c) $x^4 - 2xy^2 + 2y^3 = 32$

(d) $(x^2 + y^2)^3 = 2y^4 + 6x^2$

Ex.2) Find the tangent line to the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{36} = 1$ at the point $(-1, 4\sqrt{2})$.

Related Rates

In a related rates problem, the idea is to compute the rate of change of one quantity in terms of the rate of change of another quantity (which may be more easily measured). The procedure is to find an equation that relates the two quantities and then use the Chain Rule to differentiate both sides (or implicitly) with respect to time. In this section, we have two or more quantities that are changing with respect to time. Our first goal is to find a formula that relates the quantities. Then we will solve for the desired rate of change. If x is changing with respect to time t, then $x = x(t)$ and $\dfrac{dx}{dt}$ is the rate of change x with respect to time.

Ex.3) Suppose both x and y are changing size with respect to time t (assume x and y are both positive). At the instant when $x = 4$, the rate of change of x with respect to t is -2. If x and y are related by the equation $x^3 + 4y^2 = 84$, find the corresponding change in y at this same instant.

Ex.4) The cost equation is given by $C = 8 + 0.2x^2$, where C is the total cost when x units are produced. If the number of items manufactured is increasing at the rate of 20 per week, find the rate of change of C with respect to time when 4 items are manufactured.

Ex.5) A stone is thrown into a still pond and circular ripples move out. The radius of the disturbed region increases at a rate of 3 ft/sec. Find the rate at which the area of the disturbed region is increasing when the farthest ripple is 20 ft from the place at which the rock struck the pond.

Ex.6) A 10 ft ladder leans against a wall and slides down, with the foot of the ladder observed to be moving away from the wall at 5 ft/sec when it is 8 ft from the wall. At what speed is the top of the ladder moving downward?

Ex.7) The length of a rectangle is increasing at the rate of 2 feet per second, while the width is increasing at the rate of 1 foot per second. How fast is the area increasing when the length is 5 feet and the area is 50 squarefeet?

Chapter 6 Integration

Section 6.1 Antiderivative

Definition. Antiderivative

If $F'(x) = f(x)$, then $F(x)$ is called an antiderivative of $f(x)$.

Definition.

If F is an antiderivative of f, then the most general antiderivative of f if $F(x) + C$, where C is any arbitrary constant.

$$\frac{d}{dx}(F(x) + C) = f(x)$$

Definition. The Indefinite Integral

The collection of all antiderivatives of a function $f(x)$ is called the **indefinite integral** and is denoted by $\int f(x)\,dx$.

If we know one function $F(x)$ for which $F'(x) = f(x)$, then

$$\int f(x)\,dx = F(x) + C$$

where C is an arbitrary constant and called the **constant of integration**.

Integration Rules

(a) If $n \neq -1$, then

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$$

(b)

$$\int k\,dx = kx + C$$

(c) For any constant k,

$$\int kf(x)\,dx = k\int f(x)\,dx$$

(d)

$$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

(e)

$$\int e^x\,dx = e^x + C$$

(f)

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

Ex.8) Find the following

(a) $\int 8 \, dx$

(b) $\int x^5 \, dx$

(c) $\int \frac{1}{3}x^4 \, dx$

(d) $\int \frac{2}{t^9} \, dt$

(e) $\int (5x^4 + x^3 - 2) \, dx$

(f) $\int \frac{3}{x} \, dx$

(g) $\displaystyle\int (e^x + x^3)\ dx$

(h) $\displaystyle\int \left(3\sqrt{x} - \frac{1}{x^2} - x^{\frac{3}{2}}\right)\ dx$

(i) $\displaystyle\int \frac{4x^2 + x^3}{8x}\ dx$

(j) $\displaystyle\int (x-2)(x+3)\ dx$

Ex.9) Find y if $y(1) = 1$ and $\frac{dy}{dx} = \frac{3}{x} + \frac{1}{x^2}$

Ex.10) The daily marginal revenue function for the Black Day Sunglasses Company is given by $MR(x) = 30 - 0.0003x^2$ for $0 \leq x \leq 540$, where x represents the number of sunglasses produced and sold.

(a) Knowing that $R(50) = 1487.50$, find the revenue function.

(b) Find the price demand function for the sunglasses.

(c) What will the price be when the demand is 250 sunglasses?