MATH 142 Business Calculus

Fall 2019, WEEK 11

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Week 11 Section 6.2 6.3 Substitution, Estimating Distance Traveled

Section 6.2 Substitution

The method of substitution is based on reversing the chain rule. Recall that if $h(x) = g(f(x))$, then

$$\frac{d}{dx}(h(x)) = \frac{d}{dx}(g(f(x))) = g'(f(x)) \cdot f'(x)$$

Thus,

$$\int g'(f(x))f'(x)\ dx = g(f(x)) + C$$

Integration by u-substitution.

1. Select u (look for function where you normally have x)
2. Take the derivative of u using $\frac{du}{dx}$ notation.
3. Bring dx to the right hand side.
4. Bring any constant multiples to the left-hand side.
5. Substitute to replace all terms with x's.
6. Integrate with u's.
7. Return x's into the problem.

Ex.1) Evaluate the following.

(a) $\int 7(8x+3)^{10}\,dx$

(b) $\int \dfrac{12x}{3x^2+5}\,dx$

(c) $\int (2x^6 e^{x^7+1})\ dx$

(d) $\int 2x^2 \sqrt[4]{x^3+2}\ dx$

(e) $\displaystyle\int \frac{3(x^3+1)}{(3x^4+12x)^7}\,dx$

(f) $\displaystyle\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

(g) $\displaystyle\int \frac{x}{\sqrt{x+2}}\,dx$

(h) $\displaystyle\int x(x^5+1)^2\,dx$

(i) $\displaystyle\int \frac{(\ln x)^{44}}{x}\,dx$

(j) $\displaystyle\int \frac{4e^{2/x}}{x^2}\,dx$

Integration Formulas

(a)

$$\int [f(x)]^n f'(x)\ dx = \frac{1}{n+1}[f(x)]^{n+1} + C$$

(b)

$$\int e^{f(x)} f'(x)\ dx = e^{f(x)} + C$$

(c)

$$\int \frac{1}{f(x)} f'(x)\ dx = \ln|f(x)| + C$$

OR If $u = f(x)$, then $du = f'(x)\ dx$ and

(a)

$$\int u^n\ du = \frac{1}{n+1}u^{n+1} + C$$

(b)

$$\int e^u\ du = e^u + C$$

(c)

$$\int \frac{1}{u}\ du = \ln|u| + C$$

Section 6.3 Estimating Distance Traveled

Ex.2) Suppose a car travels at a constant 50 miles per hour for 2 hours. What is the total distance traveled?

Ex.3) Suppose a car travels 75 miles per hour for the first hour, 70 miles per hour for the second hour, and then 55 miles per hour for the last two hours of the trip. What is the total distance traveled?

Ex.4) A car starts moving at time $t = 0$ and gradually speeds up over time. Its velocity at a few particular times is shown in the table below. Estimate how far the car travels during this 12 second period.

t(seconds)	0	4	8	12
$v(t)$(ft/sec)	0	4	7	16

Ex.5) An object travels with velocity $v(t) = t^2$ where v is in feet per second and t is in seconds. Estimate how far the object traveled during the first three seconds, by using left endpoints and

(a) three rectangles of equal width.

(b) six rectangles of equal width.

Ex.6) Estimate the area under the graph of $f(x) = x - 2\ln x$ on $[1, 5]$

(a) using four approximating rectangles of equal width and right endpoints.

(b) using eight approximating rectangles of equal width and right endpoints.

Definition. The **area** of the region that lies under the graph of the continuous and positive function f is the limit of the sum of the areas of approximating rectangles.