MATH 142 Business Calculus

Fall 2019, WEEK 14

JoungDong Kim

Week 14 Section 6.6 6.7 Area Between Two Curves, Additional Applications of the Integral

Section 6.6 Area Between Two Curves

Area Between Two Curves

Let $y = f(x)$ and $y = g(x)$ be two continuous functions with $f(x) \geq g(x)$ on $[a, b]$. Then the area between the graphs of the two curves on $[a, b]$ is given by the definite integral

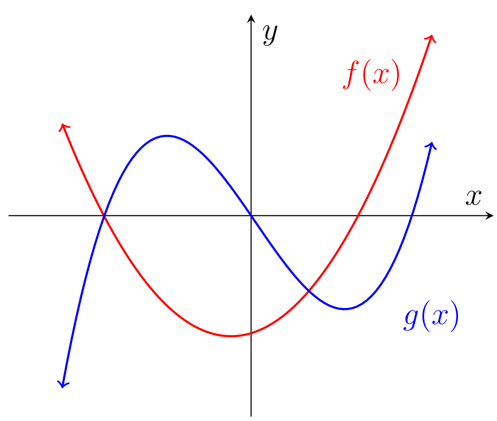
$$\int_a^b [f(x) - g(x)]\, dx$$

Ex.1) Find the area between $y = x^2 - 1$ and the x-axis on $[0, 1]$.

Ex.2) Find the area between $y = x$ and $y = x^2 + 2$ on $[-4, 3]$.

Ex.3) Find the area bounded by the curves $y = x^2 - 2x$ and $y = x$.

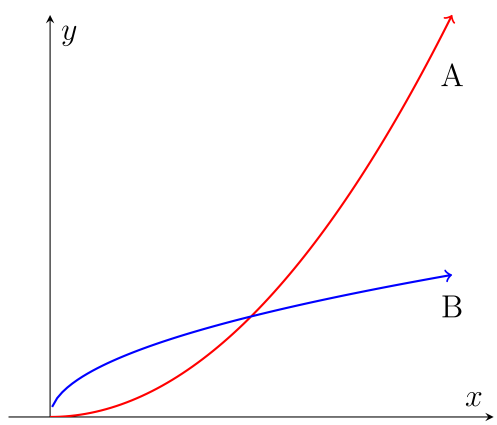
Ex.4) Given the graph below write definite integrals to represent the total area bounded by $f(x)$ and $g(x)$ on $[a, d]$.



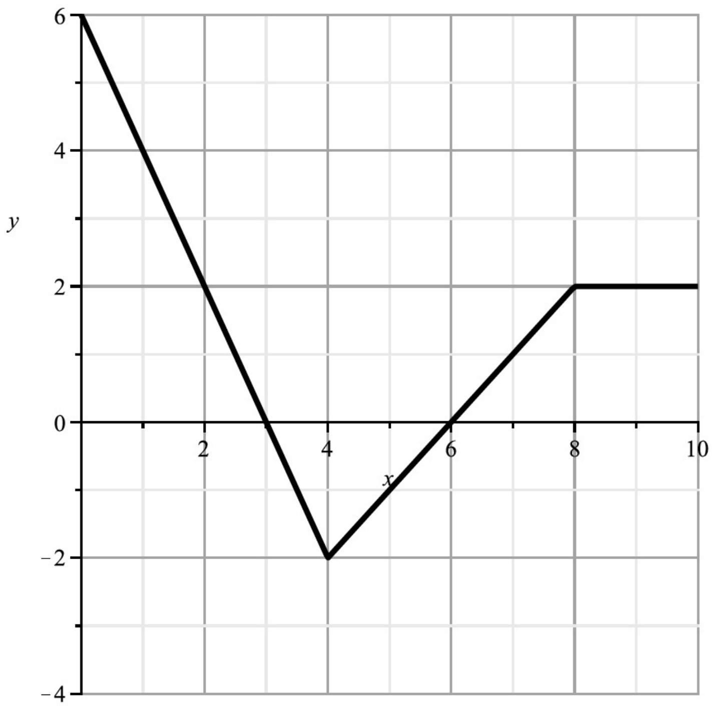
Ex.5) Find the area bounded by $y = \ln x$ and $y = 1$ on $[1, 5]$.

Ex.6) A yeast culture is growing at a rate of $w'(t) = 0.3e^{0.1t}$ grams per hour. Find the area between the graph of $w'(t)$ and t-axis on the interval $[0, 10]$ and interpret the results.

Ex.7) The figure below shows the rate of growth of two trees. If the two trees are the same height at time $t = 0$, which tree is taller after 5 years? After 10 years?



Ex.8) If $g(x) = \int_0^x f(t)dt$, where the graph of $f(t)$ is given below, where $0 \leq x \leq 10$, evaluate $g(0)$, $g(3)$, $g(6)$ and $g(10)$.



Section 6.7 Additional Applications of the Integral

Consumers' Surplus

Definition. If $p = D(x)$ is the demand equation, p_0 is the equilibrium price of the commodity, and x_0 is the equilibrium demand, then the **comsumers' surplus** is given by

$$\int_0^{x_0} [D(x) - p_0]\, dx$$

The **consumers' surplus** represents the total savings to consumers who are willing to pay more than p_0 for the product but are still able to buy the product for p_0.

Producers' Surplus

Definition. If $p = S(x)$ is the supply equation, p_0 is the equilibrium price of the commodity, and x_0 is the equilibrium demand, then the **producers' surplus** is given by

$$\int_0^{x_0} [p_0 - S(x)]\ dx$$

The **producers' surplus** represents the total gain to producers who are willing to supply units at a lower price than p_0 but are still able to supply units at p_0.

Ex.9) Find the consumers' surplus at a price level of \$150 for the price-demand equation $p = 400 - 0.5x$.

Ex.10) Find the producers' surplus when 380 items are sold, if the supply equation is given by $p = e^{0.01x}$.

Ex.11) If $p = D(x) = 80e^{-0.001x}$ and $p = S(x) = 30e^{0.001x}$, find the follwing:

(a) Equilibrium Point

(b) Consumers' surplus at the equilibrium price level.

(c) Producers' surplus at the equilibrium price level.