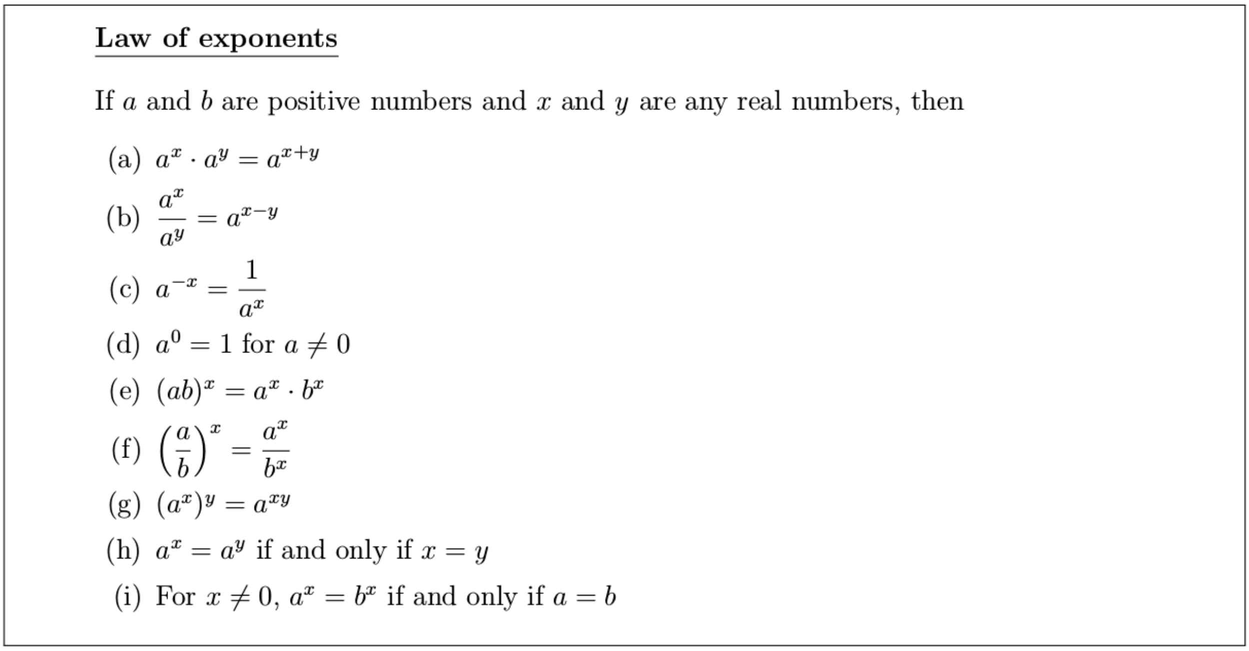


Law of exponents

If a and b are positive numbers and x and y are any real numbers, then

(a) $a^x \cdot a^y = a^{x+y}$

(b) $\dfrac{a^x}{a^y} = a^{x-y}$

(c) $a^{-x} = \dfrac{1}{a^x}$

(d) $a^0 = 1$ for $a \neq 0$

(e) $(ab)^x = a^x \cdot b^x$

(f) $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

(g) $(a^x)^y = a^{xy}$

(h) $a^x = a^y$ if and only if $x = y$

(i) For $x \neq 0$, $a^x = b^x$ if and only if $a = b$

Compound Interest

If a principal P (present value) is invested at an annual rate r (expressed as a decimal) compounded m times a year, then the amount A (future value) in the account at the end of t years is given by

$$A = P(1 + \frac{r}{m})^{mt}$$

Ex.22) If \$5,000 is invested in an account paying 2.5% compounded monthly, how much will be in the account at the end of 10 years?

$m = 12$

$$A = 5000\left(1 + \frac{0.025}{12}\right)^{12(10)}$$

$$\approx \$\ 6,418.46$$

Continuously Compounded Interest

$$A = Pe^{rt}$$

where P is principal, r is the annual interest rate compounded continuoulsy (as a decimal), t is time, and A is the accumulated amount at the end of t years.

Ex.23) What amount will an account have after five years if \$1,000 is invested at an annual rate of 3.25% compounded continuously?

$A = P \cdot e^{rt}$

$\Rightarrow A = 1000 \cdot e^{0.0325(5)}$

$\approx \$ 1,176.45$

Section 1.5 Logarithms

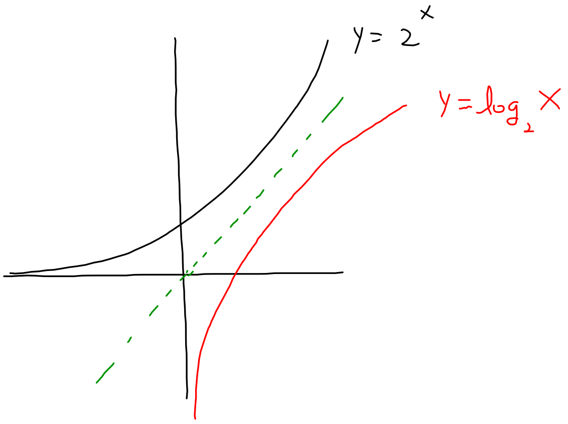
Every exponential function $f(x) = a^x$, with $a > 0$ and $a \neq 1$, is a one-to-one function and therefore has an inverse function. The inverse function f^{-1} is called the **Logarithmic function with base** a and is denoted $\log_a$.

Definition. Logarithmic function

Let a be a positive number with $a \neq 1$. The **Logarithmic function with base** a, denoted by $\log_a$, is defined by

$$y = \log_a x \iff a^y = x$$

ex) $\underset{\text{base}}{\bigcirc\!\!\!\!\!\!\;2}^{\overset{\text{exp.}}{4}} = 16 \iff \log_2 16 = 4$



Common Logarithm

The logarithm with base 10 is called the **Common logarithm** and is denoted by omitting the base

$$\log x = \log_{10} x$$

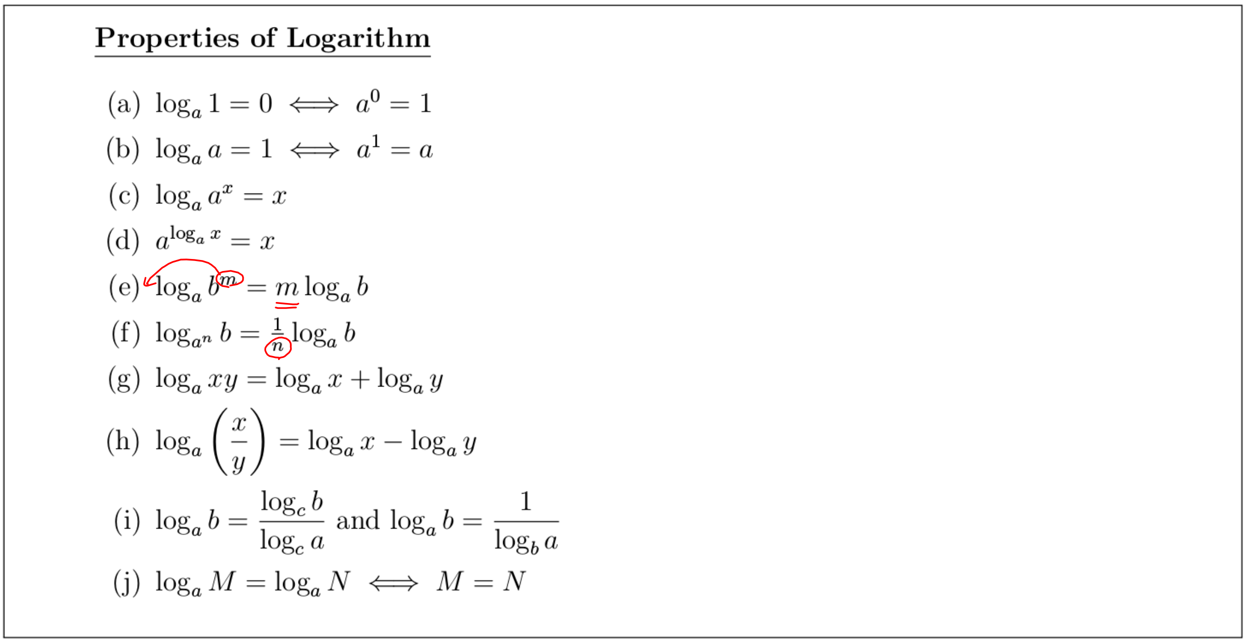
Natural Logarithm

The logarithm with base e is called the **Natural logarithm** and is denoted by ln

$$\ln x = \log_e x$$

The natural logarithm function $y = \ln x$ is the inverse function of the exponential function $y = e^x$ which is

$$\ln x = y \quad \Longleftrightarrow \quad e^y = x$$



Properties of Logarithm

(a) $\log_a 1 = 0 \iff a^0 = 1$

(b) $\log_a a = 1 \iff a^1 = a$

(c) $\log_a a^x = x$

(d) $a^{\log_a x} = x$

(e) $\log_a b^{\boxed{m}} = \underline{\underline{m}} \log_a b$

(f) $\log_{a^n} b = \frac{1}{\boxed{n}} \log_a b$

(g) $\log_a xy = \log_a x + \log_a y$

(h) $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

(i) $\log_a b = \frac{\log_c b}{\log_c a}$ and $\log_a b = \frac{1}{\log_b a}$

(j) $\log_a M = \log_a N \iff M = N$

Ex.26) Find the domain of each funtion

a) $f(x) = 2 + \log_5 \boxed{x}^{>0}$

$x > 0$

$\therefore (0, \infty)$

b) $g(x) = \log(\boxed{x - 3})^{>0}$

$x - 3 > 0$

$\therefore x > 3$

$\therefore (3, \infty)$

Ex.27) Solve the following for x:

a) $\log_b x = \frac{2}{3}\log_b 27 + 2\log_b 2 - \log_b 3$

b) $\log x + \log(x-3) = 1$

Ex.28) Find the domain of each function

$\log_a \boxed{1} = 0$

a) $f(x) = \dfrac{\sqrt[7]{2x-9}}{\log_2(4x-1)}$.

$4x - 1 > 0$

$\Rightarrow 4x > 1$

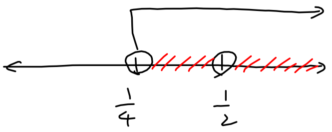
$\therefore x > \frac{1}{4}$

$\log_2(4x-1) = 0$

$\Rightarrow 4x - 1 = 1$

$\Rightarrow 4x = 2$

$\therefore x \neq \frac{1}{2}$



$\therefore \left(\frac{1}{4}, \frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$

b) $f(x) = \dfrac{\sqrt[6]{\boxed{3x+4}}^{\,\geq 0}}{\ln(\boxed{10-x})_{>0}}$.

denominator $\neq 0$

$3x + 4 \geq 0$

$\Rightarrow 3x \geq -4$

$\therefore x \geq -\frac{4}{3}$

$10 - x > 0$

$\therefore 10 > x$

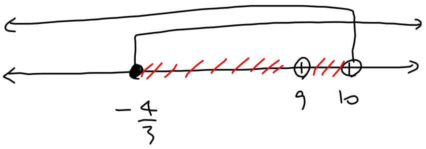
or $x < 10$

$\ln(10-x) = 0$

$\Rightarrow 10 - x = 1$

$\Rightarrow -x = -9$

$\therefore x \neq 9$



$\therefore \left[-\frac{4}{3}, 9\right), (9, 10)$

Ex.29) Find the domain of the following function.

$$f(x) = \begin{cases} \dfrac{\sqrt[3]{x^2+4x-1}}{x^2-1} & 0 \le x < 2 \\ \dfrac{e^{12x-7}}{\log_2(5-x)} & x > 2 \end{cases}$$

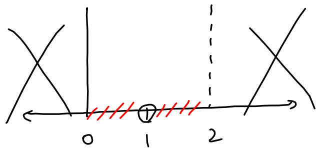
$\underline{0 \le x < 2}$

$$y = \frac{\sqrt[3]{x^2+4x-1}}{x^2-1}$$

$x^2 - 1 = 0$

$\Rightarrow \sqrt{x^2} = \pm\sqrt{1}$

$\therefore x \neq \pm 1$



$\therefore [0,1), (1,2)$

$\underline{x > 2}$

$$y = \frac{e^{12x-7}}{\log_2(\boxed{5-x})}$$

>0

$5 - x > 0$

$\therefore 5 > x$

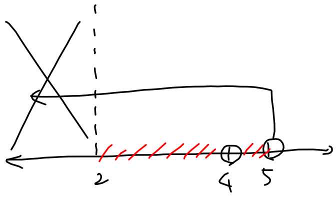
or $x < 5$

$\log_2(5-x) = 0$

$5 - x = 1$

$\Rightarrow -x = -4$

$\therefore x \neq 4$

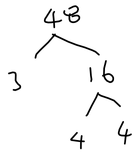


$\therefore (2,4), (4,5)$

$\therefore [0,1), (1,2), (2,4), (4,5)$

Ex.32) Given $\log_b 3 = 0.6826$ and $\log_b 4 = 0.8614$, find the following

a) $\log_b 48 = \log_b(3\cdot 4^2) = \log_b 3 + \log_b 4^{\boxed{2}}$



$= \log_b 3 + 2\cdot\log_b 4$

$= (0.6826) + 2\cdot(0.8614)$

$= 2.4054$

b) $\log_b\left(\frac{16}{b^2}\right) = \log_b 16 - \log_b b^2$

$= \log_b 4^{\boxed{2}} - \log_b b^{\boxed{2}}$

$= 2\log_b 4 - 2\overset{=1}{\boxed{\log_b b}}$

$= 2\cdot(0.8614) - 2 = -0.2772$

Ex.33) How long will it take for the amount in an account to double if the money is compounded continuously at an interest rate of 2.5% per year?

2P

$A = P \cdot e^{rt}$

$2\cancel{P} = \cancel{P} \cdot e^{0.025t}$

$\Rightarrow 2 = e^{0.025t}$

take ln on both sides

$\Rightarrow \ln 2 = \ln e^{0.025t}$

$\Rightarrow \ln 2 = 0.025t \cdot \ln e$ =1

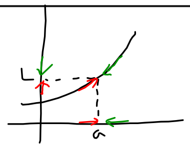
$\Rightarrow \ln 2 = 0.025t$

$\therefore t = \frac{\ln 2}{0.025} \approx 27.7259$ years

Section 3.1 Limits

Definitions

We write



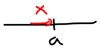
$$\lim_{x\to a} f(x) = L$$

and say "the limit of $f(x)$, as x approaches a, equals L" if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by taking x to be sufficiently close to a but not equal to a.

This says that the value of $f(x)$ get closer and closer to the number L as x gets closer and closer to the number a (from either side of a), but $x \neq a$.

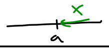
Left-Hand Limit: We write $\lim_{x\to a^-} f(x) = L$ if $f(x)$ is close to L whenever x is close to, but to the left of a.

$x < a$



Right-Hand Limit: We write $\lim_{x\to a^+} f(x) = L$ if $f(x)$ is close to L whenever x is close to, but to the right of a.

$x > a$



Existence of a Limit

If the function $f(x)$ is defined near $x = a$ but not necessarily at $x = a$, then

$$\lim_{x \to a} f(x) = L$$

if and only if both the limits

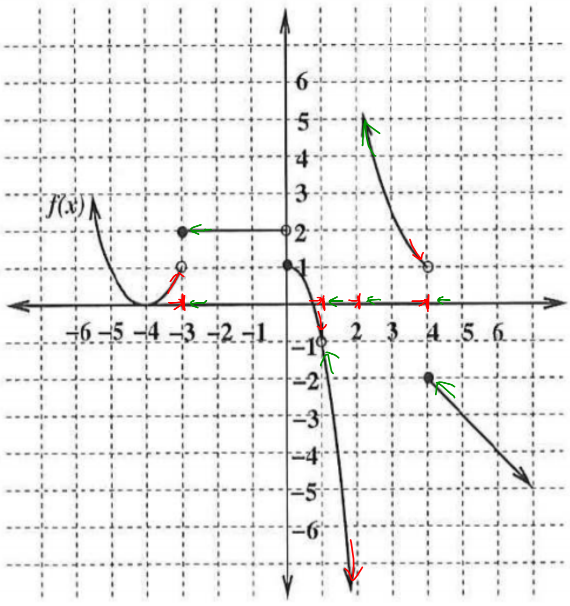
$$\lim_{x \to a^-} f(x) \quad \text{and} \quad \lim_{x \to a^+} f(x)$$

exist and are equal to the same number L.

$$\lim_{x \to a^-} f(x) = \lim_{x \to x^+} f(x)$$

$$\therefore \lim_{x \to a} f(x) \quad \text{exists}$$

Ex.1) Use the graph below to find the following limits:



a) $\lim_{x \to -3^-} f(x) =$ 1

b) $\lim_{x \to -3^+} f(x) =$ 2

c) $\lim_{x \to -3} f(x) =$ DNE

d) $\lim_{x \to 1^-} f(x) =$ -1

e) $\lim_{x \to 1^+} f(x) =$ -1

f) $\lim_{x \to 1} f(x) =$ -1

g) $\lim_{x \to 4^-} f(x) =$ 1

h) $\lim_{x \to 4^+} f(x) =$ -2

i) $\lim_{x \to 4} f(x) =$ DNE

j) $\lim_{x \to 2^-} f(x) =$ $-\infty$

k) $\lim_{x \to 2^+} f(x) =$ $+\infty$

l) $\lim_{x \to 2} f(x) =$ DNE