MATH 142 Business Calculus

Fall 2019, WEEK 4

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Week 4 Section 4.1, 4.2 Simple Derivative Rules and Marginal Analysis, Product and Quotient Rules

Chapter 4 Rules for the Derivative

Section 4.1 Derivatives of Powers, Exponents, and Sums

Differentiation Formulas

1. **Constant rule:** If $f(x) = c$ where c is any constant then

$$f'(x) = 0$$

2. **Power Rule:** If $f(x) = x^n$, where n is a real number, then

$$f'(x) = nx^{n-1}$$

3. **Constant times a function rule:**

$$(cf(x))' = cf'(x)$$

4. **Sum/Difference rule:** If $f(x) = g(x) \pm h(x)$, then

$$f'(x) = g'(x) \pm h'(x)$$

Ex.1) Differentiate the following functions:

a) $f(x) = 5$

b) $f(x) = \pi$

c) $f(x) = e\sqrt{2}$

d) $f(x) = x^7$

e) $f(x) = x^{\frac{1}{2}}$

f) $f(x) = \sqrt[3]{x^5}$

g) $f(x) = \dfrac{5}{x^2}$

h) $y = 2x^3 + 5x - 9$

i) $g(t) = \sqrt{3}t - t^{\frac{1}{2}} + e$

j) $h(x) = \dfrac{x^3 + 2x^2 - 2x + 3}{x}$

k) $y = \dfrac{x^4 + 4x^2}{3\sqrt{x}}$

Ex.2) If JD drop the ball from a building 400 feet tall, its height above the ground (in feet) after t seconds is given by $s(t) = 400 - 16t^2$

a) Compute $s'(t)$

b) Compute $s(2)$ and $s'(2)$

Ex.3) If $f(x) = 3x^4 - 2x^2$, where does the graph of the function have a horizontal tangent line?

Ex.4) Suppose the total cost (in dollars) of producing x books is given by

$$C(x) = 0.5x^2 - 12x + 100$$

a) Find $C(15) - C(14)$

b) Find $C'(14)$

Marginal Business Functions

Approximate change in the dependent variable (cost, revenue, profit) when the independent variable (the number of items produced/sold) is changed by a single unit.

- **Marginal Cost Function**

$$MC(x) = C'(x)$$

- **Marginal Revenue Function**

$$MR(x) = R'(x)$$

- **Marginal Profit Function**

$$MP(x) = P'(x)$$

NOTE. The marginal functions approximate the Cost/Revenue/Profit of the next item.

Ex.5) The total profit (in dollars) of producing x ski jackets is given by

$$P(x) = -0.2x^2 + 176x - 21900$$

a) Find the exact profit realized from the sale of the 201st ski jacket.

b) Use the marginal profit function to approximate the profit realized from the sale of the 201st ski jacket.

Derivatives of Exponential and Logarithmic Functions

(a) If $f(x) = e^x$ then
$$f'(x) = e^x$$

(b) If $f(x) = \ln x$ then
$$f'(x) = \frac{1}{x}$$

(c) If $f(x) = b^x$ then
$$f'(x) = b^x \cdot \ln b$$

(d) If $f(x) = \log_b x$ then
$$f'(x) = \frac{1}{x \ln b}$$

Ex.6) Find the derivative of each of the following functions:

a) $f(x) = 7e^x$

b) $f(x) = 2(3)^x$

c) $f(x) = \ln x + 3$

d) $f(x) = 4x^2 - \sqrt[3]{x} + \log_7 x^3 - 5^x$

e) $y = \ln(x^7) + 3(2)^x$

Ex.7) Find the equation of the line tangent to the graph of $f(x) = e^x + \ln x$ at $x = 1$.

Section 4.2 Derivatives of Products and Quotients

Product Rule

If $h(x) = f(x) \cdot g(x)$ and if $f'(x)$ and $g'(x)$ exist, then

$$h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Quotient Rule

If $h(x) = \dfrac{f(x)}{g(x)}$ and if $f'(x)$ and $g'(x)$ exist, then

$$h'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

Ex.8) Find the derivative of the following functions:

a) $h(x) = x^2(x^2 + 4x)$

b) $h(x) = (x^2 + 3)(\sqrt[4]{x} + \sqrt[8]{x^3})$

c) $h(x) = \dfrac{x^2+5}{3x}$

d) $h(x) = \dfrac{3\sqrt{x}+7x}{x^2-4x+\frac{1}{x}}$

e) $f(x) = 5x^4e^x$

f) $g(x) = \dfrac{x^2e^x + 5}{7 - e^x}$

g) $h(x) = \dfrac{2^x + 5x^2}{\log_2 x^4 - \ln x}$

Ex.9) Find the equation of the tangent line to $f(x) = \dfrac{x^2+1}{3x^3-4x^2+2}$ at $x = 2$.

Ex.10) Suppose that $f(2) = -1$, $g(2) = 3$, $f'(2) = -4$, and $g'(2) = 6$. Find $h'(2)$ for each of the following:

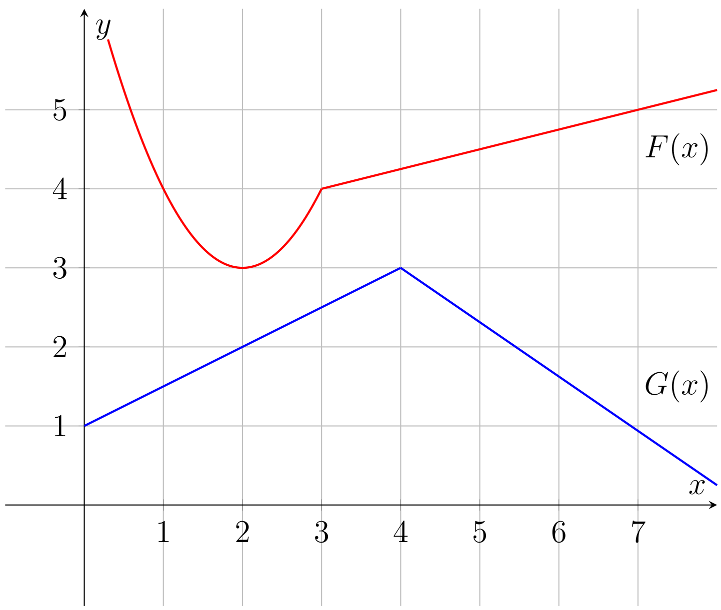
a) $h(x) = 2f(x) - 3g(x)$

b) $h(x) = f(x)g(x)$

c) $h(x) = \dfrac{f(x)}{g(x)}$

d) $h(x) = \dfrac{f(x)}{1 + g(x)}$

Ex.11) Let $P(x) = F(x)G(x)$ and $Q(x) = \dfrac{F(x)}{G(x)}$, where F and G are the functions whose graphs are shown below.



a) Find $P'(2)$

b) Find $Q'(7)$