Ex.3) Suppose $w(x) = u(v(x))$ and $u(0) = 1$, $v(0) = 2$, $u'(0) = 3$, $u'(2) = 4$, $v'(0) = 5$, and $v'(2) = 6$. Find $w'(0)$.

$$w'(x) = u'(v(x)) \cdot v'(x)$$

$$\Rightarrow w'(0) = u'(\underbrace{v(0)}_{=2}) \cdot \underbrace{v'(0)}_{=5}$$

$$= \underbrace{u'(2)}_{=4} \cdot 5$$

$$= 4 \cdot 5 = 20$$

Ex.4) Let $y = \ln u$ and $u = 5x^4 + x^6$. Find $\frac{dy}{dx}$.

$y(u)$ $\quad$ $u(x)$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{u} \cdot (20x^3 + 6x^5)$$

$$= \frac{1}{(5x^4 + x^6)} (20x^3 + 6x^5)$$

or

$$y(u) = \ln u$$

$$\Rightarrow y(x) = \ln(5x^4 + x^6)$$

$$\frac{dy}{dx} = y'(x) = \frac{1}{(5x^4 + x^6)} \cdot (20x^3 + 6x^5)$$

Ex.5) Keith invests \$5,000 into a savings account offering interest at an annual rate of 2.4% compounded continuously. How fast is the balance growing after 8 years?

$A = P \cdot e^{rt}$ $\qquad A'(8)$

$\Rightarrow A(t) = 5000 \cdot e^{0.024t}$

$\frac{dA}{dt} = A'(t) = 5000 \cdot e^{(0.024t)} \cdot (0.024)$

$\Rightarrow A'(8) = 5000 \cdot e^{(0.024(8))} \cdot (0.024)$

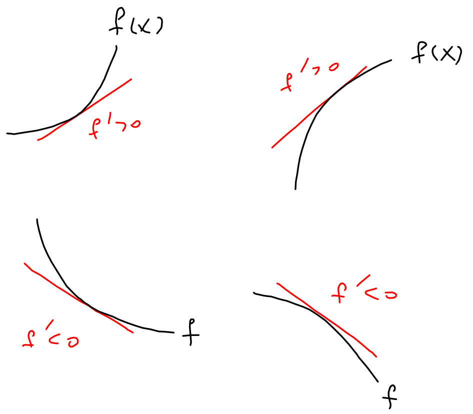
$= \$145.40$ /year

Chapter 5 Curve Sketching and Optimization

Section 5.1 The First Derivative

Test for Increasing or Decreasing Functions

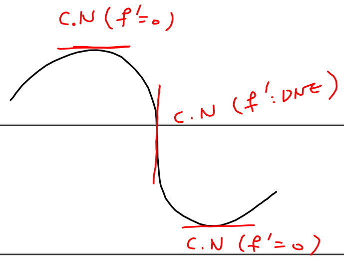
- If for all $x \in (a, b)$, $f'(x) > 0$, then $f(x)$ is increasing($\nearrow$) on (a, b)
- If for all $x \in (a, b)$, $f'(x) < 0$, then $f(x)$ is decreasing($\searrow$) on (a, b)



Definition. Critical Value: A value $x = c$ is a **critical value** for a function $f(x)$ if

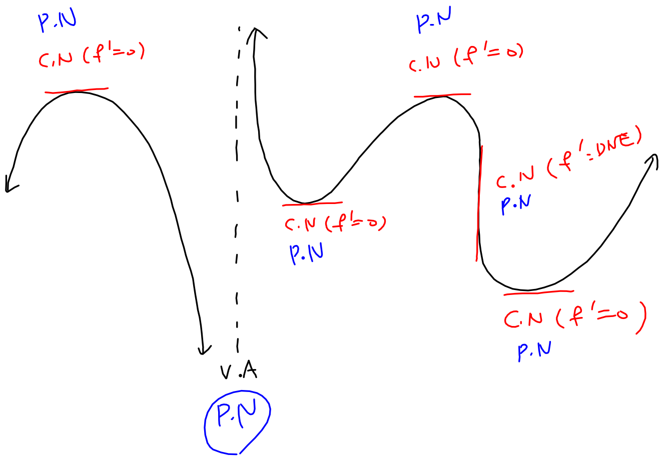
(a) c is in the domain of the function $f(x)$ and

(b) $f'(c) = 0$ or $f'(x)$ does not exist.



Definition. Partition number: A **partition number** of $f'(x)$ is a value of x such that $f'(x) = 0$ or $f'(x)$ is undefined. ↖ C.N + undefined

ex)



Ex.6) Find the critical values and partition numbers for the following functions and then determine where the function is increasing/decreasing.

a) $f(x) = -x^3 + 12x - 5$

Step 1) Find C.N

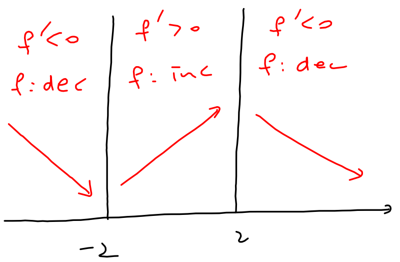
$f'(x) = -3x^2 + 12$

$= -3(x^2 - 4)$

$= -3(x-2)(x+2) = 0$

$\therefore$ $x = 2, -2$: C.Ns , P.Ns

Step 2) Sign chart for f'



$\therefore$ dec. $(-\infty, -2)$, $(2, \infty)$

inc. $(-2, 2)$

local min at $x = -2$

local Max at $x = 2$

b) $g(x) = \dfrac{x+3}{5-x}$ Domain: $\mathbb{R}$ except $x=5$

$\therefore$ $x=5$: P.N

Step 1) C.N

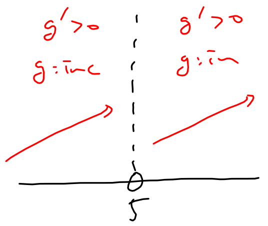
$$g'(x) = \frac{(1)\cdot(5-x) - (x+3)\cdot(-1)}{(5-x)^2}$$

$$= \frac{5-x+x+3}{(5-x)^2}$$

$$= \frac{8}{(5-x)^2} = 0$$

$\therefore$ No C.N $\therefore$ P.N : $x=5$

Step 2) Sign chart for g'



$\therefore$ inc. $(-\infty, 5), (5, \infty)$

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c) $h(x) = x \ln x - x$ Domain: $x > 0$

Step 1) C.N

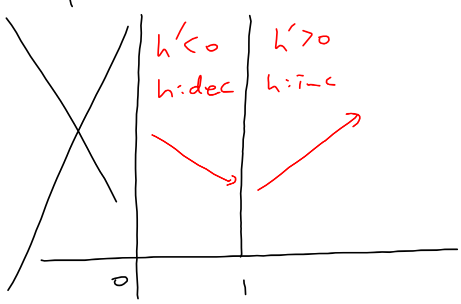
$$h'(x) = 1\cdot \ln x + x\cdot\frac{1}{x} - 1$$

$$= \ln x + 1 - 1$$

$$= \ln x = 0$$

$\therefore$ $x = 1$: C.N, P.N

Step 2) Sign chart for h'



$\therefore$ dec.: $(0, 1)$

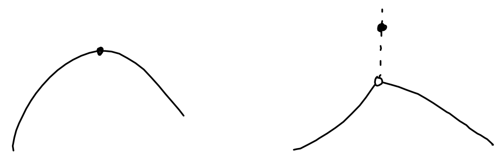
inc.: $(1, \infty)$

local min at $x=1$

Definition. Relative Maximum and Relative Minimum:

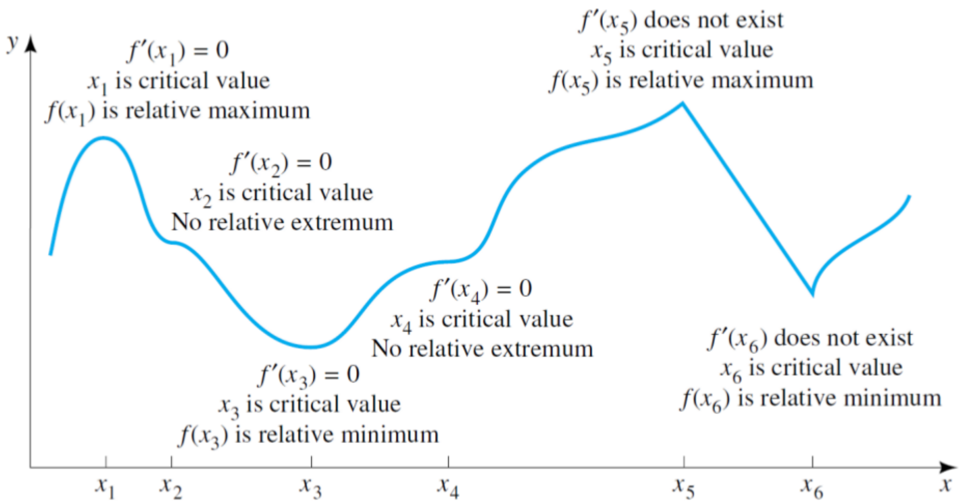
- We say that the quantity $f(c)$ is a **relative (local) maximum** if $f(x) \leq f(c)$ for all x in some open interval (a, b) that contains c.
- We say that the quantity $f(c)$ is a **relative (local) minimum** if $f(x) \geq f(c)$ for all x in some open interval (a, b) that contains c.

Local Max :



Local min :

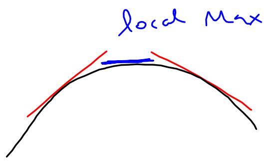
Definition. Relative Extremum We say that $f(c)$ is a **relative (local) extremum** if $f(c)$ is a relative maximum or a relative minimum.



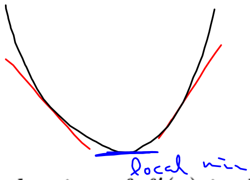
First Derivative Test

Suppose f is defined on (a, b) and c is a critical value in the interval (a, b).

(a) If $f'(x) > 0$ for x near and to the left of c and $f'(x) < 0$ for x near and to the right of c, then we have $\nearrow\searrow$ and $f(c)$ is a relative maximum.



(b) If $f'(x) < 0$ for x near and to the left of c and $f'(x) > 0$ for x near and to the right of c, then we have $\searrow\nearrow$ and $f(c)$ is a relative minimum.



(c) If the sign of $f'(x)$ is the same on both sides of c, then $f(c)$ is not a relative extremum.

Ex.7) Determine the intervals where the following functions are increasing and decreasing and find the local extrema.

a) $f(x) = x^4 + 2x^3 + 5$

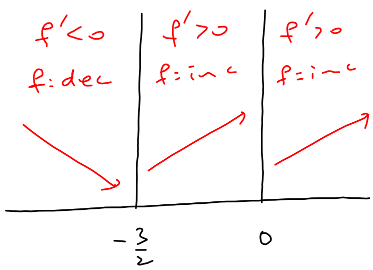
Step 1) Find C.N

$f'(x) = 4x^3 + 6x^2$

$= 2x^2(2x+3) = 0$

$\therefore\ x = 0, -\frac{3}{2}$: C.N,

Step 2) Sign chart for f'



$\therefore$ dec. : $(-\infty, -\frac{3}{2})$

inc. : $(-\frac{3}{2}, 0), (0, \infty)$

local min : $\left(-\frac{3}{2}, f\left(-\frac{3}{2}\right)\right)$

$= \left(-\frac{3}{2}, \frac{53}{16}\right)$

b) $g(x) = \dfrac{2}{x^2 - 16} = 2(x^2-16)^{-1}$

Domain: $\mathbb{R}$ except: $x = 4, -4$: P.Ns

Step 1) C.N

$g'(x) = -2(x^2-16)^{-2} \cdot (2x)$

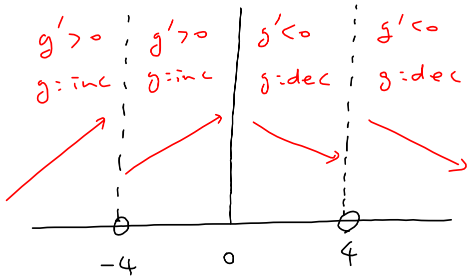
$= \dfrac{-4x}{(x^2-16)^2} = 0$

$\Rightarrow -4x = 0$

$\therefore x = 0$: C.N

$\therefore$ P.Ns: $x = 0, 4, -4$

Step 2) Sign chart for g'



$\therefore$ inc: $(-\infty, -4), (-4, 0)$

dec: $(0, 4), (4, \infty)$

local Max: $(0, f(0))$

$= (0, -\frac{1}{8})$

c) $h(x) = (x+2)e^x$