MATH 142 Business Calculus

Fall 2019, WEEK 6

JoungDong Kim

Week 6 Section 4.3, 4.4, 5.1 Chain Rule, Derivatives of Exponential and Logarithmic Functions, Analyzing Graphs with the First Derivative

Section 4.3 The Chain Rule

The Chain Rule: If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F(x) = f(g(x))$ is differentiable at x and is given by

$$F'(x) = f'(g(x)) \cdot g'(x)$$

In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

General Derivative Rules

- If $y = [f(x)]^n$ then
$$y' = n[f(x)]^{n-1} \cdot f'(x)$$
- If $y = e^{f(x)}$ then
$$y' = e^{f(x)} \cdot f'(x)$$
- If $y = \ln(f(x))$ then
$$y' = \frac{1}{f(x)} \cdot f'(x)$$
- If $y = b^{f(x)}$ then
$$y' = b^{f(x)} \cdot \ln b \cdot f'(x)$$
- If $y = \log_b f(x)$ then
$$y' = \frac{1}{f(x) \cdot \ln b} \cdot f'(x)$$

Ex.1) Differentiate the following:

a) $f(x) = (4x^2 + 7x)^5$

b) $g(x) = 6(x^{\frac{1}{2}} - 3x)^4$

c) $y = \dfrac{3}{(t^2 + 3t + 4)^4}$

d) $F(x) = e^{x^2}$

e) $H(x) = 3x^5 e^{x^4}$

f) $h(x) = \dfrac{3}{\sqrt[4]{(3x+2)^5}}$

g) $f(x) = (4x^2 + 5)^6(\sqrt[3]{(3x^4 - 5x + 7)^4})$

h) $y = \log_8(\sqrt{1 + x^2} + 10x)$

i) $y = \log_6 \left(\dfrac{x - 1}{x + 2} \right)$

j) $y = \dfrac{5 \ln((x^2 + x)^5)}{x^3}$

k) $y = \ln \left(\dfrac{x^4 \cdot (x^2 + 2)^3}{(2x + 4)^2} \right)$

l) $y = x^3 \cdot 5^{x^4 + 2}$

Ex.2) Find the value(s) of x where the tangent line is horizontal for $f(x) = \dfrac{x^2}{(2-3x)^3}$

Ex.3) Suppose $w(x) = u(v(x))$ and $u(0) = 1$, $v(0) = 2$, $u'(0) = 3$, $u'(2) = 4$, $v'(0) = 5$, and $v'(2) = 6$. Find $w'(0)$.

Ex.4) Let $y = \ln u$ and $u = 5x^4 + x^6$. Find $\dfrac{dy}{dx}$.

Ex.5) Keith invests \$5,000 into a savings account offering interest at an annual rate of 2.4% compounded continuously. How fast is the balance growing after 8 years?

Chapter 5 Curve Sketching and Optimization

Section 5.1 The First Derivative

Test for Increasing or Decreasing Functions

- If for all $x \in (a, b)$, $f'(x) > 0$, then $f(x)$ is increasing($\nearrow$) on (a, b)
- If for all $x \in (a, b)$, $f'(x) < 0$, then $f(x)$ is decreasing($\searrow$) on (a, b)

Definition. Critical Value: A value $x = c$ is a **critical value** for a function $f(x)$ if

(a) c is in the domain of the function $f(x)$ and

(b) $f'(c) = 0$ or $f'(x)$ does not exist.

Definition. Partition number: A **partition number** of $f'(x)$ is a value of x such that $f'(x) = 0$ or $f'(x)$ is undefined.

Ex.6) Find the critical values and partition numbers for the following functions and then determine where the function is increasing/decreasing.

a) $f(x) = -x^3 + 12x - 5$

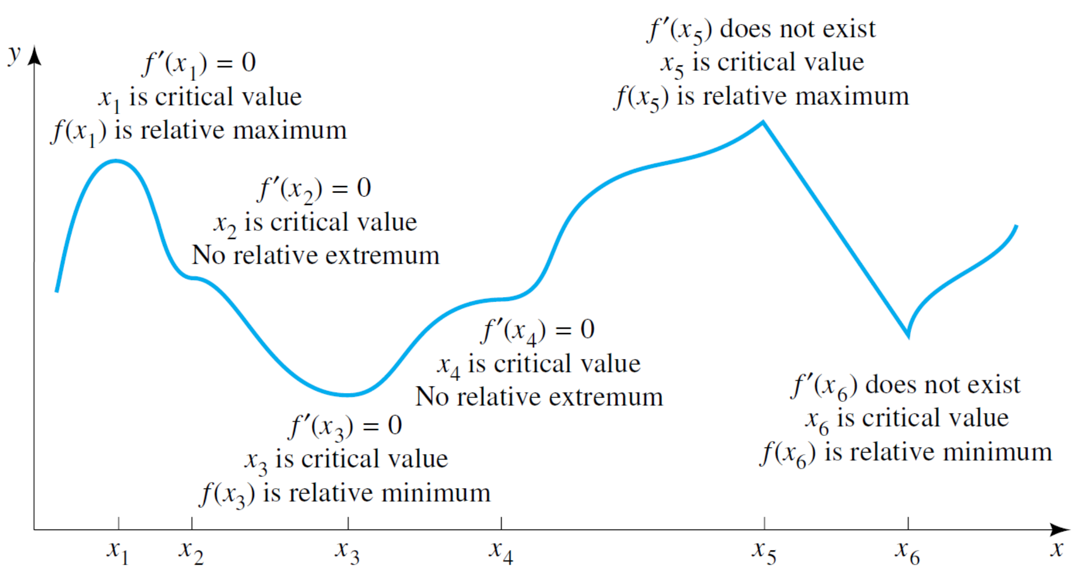
b) $g(x) = \dfrac{x+3}{5-x}$

c) $h(x) = x \ln x - x$

Definition. Relative Maximum and Relative Minimum:

- We say that the quantity $f(c)$ is a **relative (local) maximum** if $f(x) \leq f(c)$ for all x in some open interval (a, b) that contains c.
- We say that the quantity $f(c)$ is a **relative (local) minimum** if $f(x) \geq f(c)$ for all x in some open interval (a, b) that contains c.

Definition. Relative Extremum We say that $f(c)$ is a **relative (local) extremum** if $f(c)$ is a relative maximum or a relative minimum.



First Derivative Test

Suppose f is defined on (a, b) and c is a critical value in the interval (a, b).

(a) If $f'(x) > 0$ for x near and to the left of c and $f'(x) < 0$ for x near and to the right of c, then we have $\nearrow\searrow$ and $f(c)$ is a relative maximum.

(b) If $f'(x) < 0$ for x near and to the left of c and $f'(x) > 0$ for x near and to the right of c, then we have $\searrow\nearrow$ and $f(c)$ is a relative minimum.

(c) If the sign of $f'(x)$ is the same on both sides of c, then $f(c)$ is not a relative extremum.

Ex.7) Determine the intervals where the following functions are increasing and decreasing and find the local extrema.

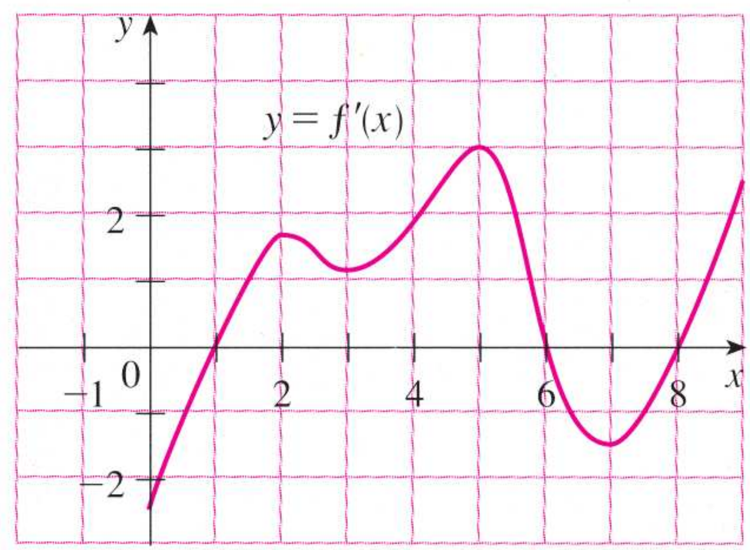
a) $f(x) = x^4 + 2x^3 + 5$

b) $g(x) = \dfrac{2}{x^2 - 16}$

c) $h(x) = (x + 2)e^x$

d) $f'(x) = (x+2)^5(x-3)^7e^{(x-6)}$

Ex.8) The graph of the derivative $f'(x)$ is shown below.



a) on what intervals is f increasing?

b) on what intervals is f decreasing?

c) the critical value(s).

d) At what values of x does $f(x)$ have a local maximum or minimum?