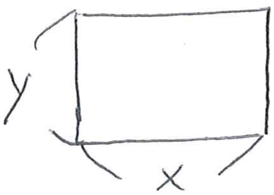
Ex 4)

Q: Find min. perimeter.



$\therefore P = 2x + 2y$

since area is 225

$\therefore x \cdot y = 225$

$\Rightarrow y = \dfrac{225}{x}$

Then $P(x) = 2x + 2\left(\dfrac{225}{x}\right)$

$\therefore P(x) = 2x + \dfrac{450}{x} = 2x + 450x^{-1}$

Find min of P

C.N: $P'(x) = 2 - 450x^{-2}$ $\quad \times x^2 \quad \times x^2$

$= 2 - \dfrac{450}{x^2} = 0$

$\Rightarrow 2x^2 - 450 = 0$

$\Rightarrow x^2 - 225 = 0$

$\Rightarrow (x+15)(x-15) = 0$

$\therefore x = \cancel{-15}, 15$: C.Ns

By Second Deriv. Test

$P''(x) = 900x^{-3} = \dfrac{900}{x^3}$

$\Rightarrow P''(15) = \dfrac{900}{(15)^3} > 0 : \cup$

$\therefore$ P has a local min at $x = 15$

$\therefore$ Dimensions for min perimeter are

15 cm x 15cm

Ex 5) point: (#, unit price)

Q: Max Revenue

Demand price : $p = mx + b$

$(1{,}600, 2.40)$, $(1650, 2.35)$

Then slope $m = \dfrac{P_2 - P_1}{X_2 - X_1} = \dfrac{2.35 - 2.40}{1650 - 1600}$

$= \dfrac{-0.05}{50} = -0.001$

$p - p_1 = m(x - x_1)$

$\Rightarrow p - 2.40 = -0.001(x - 1600)$

$\Rightarrow p - 2.40 = -0.001x + 1.6$

$\boxed{\therefore p = -0.001x + 4}$

Then Revenue : $R(x) = p \cdot x = (-0.001x + 4) \cdot x$

$\therefore R(x) = -0.001x^2 + 4x$

Find Max of R

C.N: $R' = -0.002x + 4 = 0$

$\Rightarrow -0.002x = -4$

$\therefore x = \dfrac{-4}{-0.002} = 2000$

Use the second Deriv. Test

$R'' = -0.002$

$\Rightarrow R''(2000) = -0.002 < 0 : \frown$

$\therefore$ R has a local Max at $x = 2000$.

$\therefore$ unit price for Max. Revenue is $p(2000) = -0.001(2000) + 4$
$= \$2$

Ex6) Q: Max profit. $P(x) = R(x) - C(x)$

- Revenue : $R(x) = p \cdot x$
- demand price : $p = mx + b$

$(300, 80)$, $(297, 81)$

$\Rightarrow$ slope : $m = \dfrac{p_2 - p_1}{x_2 - x_1} = \dfrac{81 - 80}{297 - 300} = \dfrac{1}{-3}$

Then $p - p_1 = m(x - x_1)$

$\Rightarrow p - 80 = -\frac{1}{3}(x - 300)$

$\Rightarrow p - 80 = -\frac{1}{3}x + 100$

$\therefore p = -\frac{1}{3}x + 180$

$\therefore R(x) = p \cdot x = \left(-\frac{1}{3}x + 180\right) \cdot x$

$\therefore R(x) = -\frac{1}{3}x^2 + 180x$

- Cost : $C(x) = 10 \cdot x$

Then

Profit : $P(x) = R(x) - C(x)$

$= \left(-\frac{1}{3}x^2 + 180x\right) - (10x)$

$\therefore P(x) = -\frac{1}{3}x^2 + 170x$

Find Max. of P

C.N : $P' = -\frac{2}{3}x + 170 = 0$

$\Rightarrow -\frac{2}{3}x = -170$

$\Rightarrow x = -170 \cdot \frac{-3}{2}$

$\therefore x = 255$: C.N

By Second Deriv. test.

$P'' = -\frac{2}{3}$

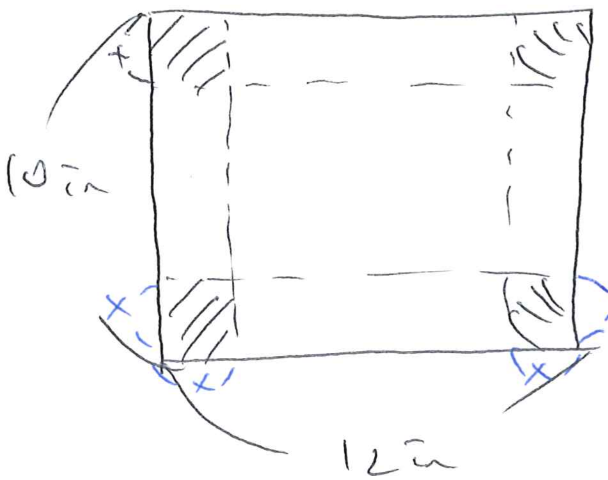
$\Rightarrow P''(255) = -\frac{2}{3} < 0 : \frown$

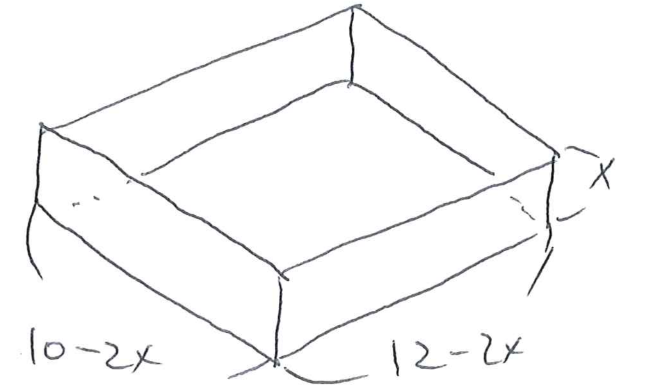
$\therefore$ P has a local Max at $x = 255$

$\therefore$ Max profit : $P(255) = -\frac{1}{3}(255)^2 + 170(255)$

$= \$21675$

Ex 7) Q: Max. Volume





Volume : $V = (10-2x)(12-2x)\cdot x$

$= (120 - 20x - 24x + 4x^2)\cdot x$

$\therefore V(x) = 120x - 44x^2 + 4x^3$

Find Max of V

C.N : $V' = 120 - 88x + 12x^2 = 0$

$\Rightarrow 3x^2 - 22x + 30 = 0$

Use the Quadratic formular.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-22) \pm \sqrt{(-22)^2 - 4\cdot(3)(30)}}{2(3)}$$

$$= \frac{22 \pm 2\sqrt{31}}{6} \approx \cancel{5.5226} \text{ or } 1.8107 \text{ : C.Ns}$$

By Second Deriv. Test

$V'' = -88 + 24x$

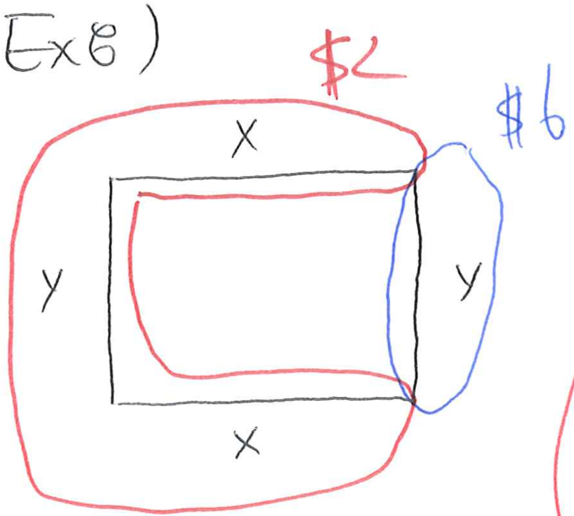
$\Rightarrow V''(1.8107) = -88 + 24(1.8107) < 0$

$\Rightarrow$ V has a local Max at $x = 1.8107$

$\therefore$ Dimensions for Max volume are

6.3786 in × 8.3786 in × 1.8107 in

Ex8)



Q: Max. area

Area : $A = x \cdot y$

price

$: \$2(2x+y) + \$6 \cdot y = 320$

$\Rightarrow 4x + 2y + 6y = 320$

$\Rightarrow 4x + 8y = 320$

$\Rightarrow 8y = 320 - 4x$

$\therefore y = 40 - \frac{1}{2}x$

Then

$$A(x) = x \cdot \left(40 - \frac{1}{2}x\right)$$

$$\therefore A(x) = 40x - \frac{1}{2}x^2$$

Find Max of A

C.N. : $A' = 40 - x = 0$

$\therefore x = 40$: C.N

By Second Deriv. Test.

$A'' = -1$

$\to A''(40) = -1 < 0$: $\cap$

$\therefore$ A has a local Max at $x = 40$

$\therefore$ Dimensions for Max. area are

40 ft x 20 ft.