MATH 142 Business Calculus

Fall 2019, WEEK 8

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Week 8 Section 5.5 5.6 Absolute Extrema, Optimization and Modeling

Section 5.5 Absolute Extrema

Definition. Maximum, Minimum. A function f has an **absolute maximum** at c if $f(c) \geq f(x)$ for all x in the domain of f. A function f has an **absoulte minimum** at c if $f(c) \leq f(x)$ for all x in the domain of f. In this case, we call $f(c)$ the maximum value or minimum value, repectively.

cf) A function f has an **local maximum** at c if $f(c) \geq f(x)$ when x is near c. A function f has an **local minimum** at c if $f(c) \leq f(x)$ when x is near c.

Extreme Value Theorem. A function f that is **continuous** on a **closed interval** $[a, b]$ has both an absolute maximum value and an absolute minimum value on that interval.

Finding Absolute Extrema on a Closed Interval

1. Check to make certain that f is continuous over $[a, b]$.
2. Find the critical values in the interval (a, b).
3. Evaluate f at the endpoints a and b and at the critical values found in step 2.
4. The largest value obtained from the previous step is the absolute maximum of $f(x)$ on $[a, b]$ and the smallest value obtained is the absolute minimum of $f(x)$ on $[a, b]$.

Ex.1) Find the absolute extrema of the following functions on the given intervals.

a) $f(x) = 2x^3 - 3x^2 - 12x + 24$ on $[1, 4]$

b) $g(x) = x + \dfrac{4}{x}$ on $[1, 8]$

c) $h(x) = x^3 + 3x^2 - 9x - 7$ on $[-4, 0)$

Ex.2) Find the absolute extrema of each function on the given interval.

a) $f(x) = 6x - x^2 + 4$ on $(0, \infty)$

b) $g(x) = 2x + \dfrac{8}{x}$ on $(0, 10)$

Section 5.6 Optimization and Modeling

Strategy for solving Optimization Problems

(a) Introduce variables, look for relationships among these variables, and construct a mathematical model of the form.

(b) Find the critical values of $f(x)$.

(c) Use the procedures developed in Section 5.5 to find the absolute maximum (or minimum) value of $f(x)$ on the interval I and the value(s) of x where this occurs.

(d) Use the solution to answer all questions asked in the problem.

Ex.3) Find two positive numbers whose sum is 60 and whose product is a maximum.

Ex.4) Find the dimensions of a rectangle of area 225 square centimeters that has the smallest perimeter. What is the perimeter?

Ex.5) A university student center sells 1,600 cups of coffee per day at a price of \$2.40. A market survey shows that for every \$0.05 reduction in price 50 more cups of coffee will be sold. How much should the student center charge for a cup of coffee in order to maximize its revenue?

Ex.6) A 300-room hotel in Las Vegas is filled to capacity every night at \$80 a room. For each \$1 increase in rent, 3 fewer rooms are rented. If each rented room costs \$10 to service per day, how much should the management charge for each room to maximize gross profit? What is the maximum gross profit?

Ex.7) An open-top box used to carry small toys is to be made from a 10 inch by 12 inch piece of cardboard by cutting identical squares from the corners and then folding up the flaps. Determine the dimensions of the box to maximize the volume.

Ex.8) A homeowner has \$320 to spend on building a fence around a rectangular garden. Three sides of the fence will be constructed with wire fencing at a cost \$2 per foot. In order to provide a view block for a neighbor, the fourth side is to be constructed with wood fencing at a cost of \$6 per foot. Find the dimensions of the garden with largest area that can be enclosed with \$320 worth of fencing.