Math 622 Practice question for test 2

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1. Let $M^2 \subset \mathbb{E}^3$ be a regular oriented surface. Define a new surface $\tilde{M}$ whose points are $y(x) = x + ae_3(x)$, where $x \in M$, e_3 is the unit normal to M at x, and a is a constant. Compute the Gauss and mean curvature functions of $\tilde{M}$ at $y(x)$ in terms of those of M at x.

2. Consider the special case where M has constant mean curvature $c \neq 0$. Determine a value of a such that the resulting $\tilde{M}$ has constant Gauss curvature.

3. Let $M^2 \subset \mathbb{E}^3$ be a regular oriented surface without umbillic points. Show that M has zero mean curvature iff the Gauss map is conformal, i.e., iff for all $p \in M$ and for all $v, w \in T_pM$, that $\langle de_3(v), de_3(w)\rangle_{e_3(p)} = \lambda(p)\langle v, w\rangle_p$ for some non-vanishing function $\lambda : M \to \mathbb{R}$.

4. CFB 1.8.4.3