Chapter 2 Homework Solutions
Compiled by Joe Kahlig

---

1. x = the number of dimes
   y = the number of quarters

   $x + y = 32$
   $.1x + .25y = 5.15$

   Solution:
   19 dimes
   13 quarters

2. x = the number of nickels
   y = the number of quarters

   $x + y = 150$
   $.05x + .25y = 18.50$

   Solution:
   95 nickels
   55 quarters

3. x = amount invested in Fund A
   y = amount invested in Fund B

   $x + y = 50000$
   $.074x + .098y = 4072$

   Solution:
   $34,500 invested in Fund A
   $15,500 invested in Fund B

4. x = number of five dollar bills
   y = number of ten dollar bills
   z = number of twenty dollar bills

   $x + y + z = 70$
   $5x + 10y + 20z = 740$
   $x - 3y = 0$

   Solution:
   36 five dollar bills
   12 ten dollar bills
   22 twenty dollar bills

5. x = number of minutes jogging
   y = number of minutes playing handball
   z = number of minutes riding a bike

   $x + y + z = 60$
   $11x + 13y + 5z = 660$
   $x - 2z = 0$

   Solution:
   20 minutes jogging
   30 minutes playing handball
   10 minutes riding a bike

6. x = number of hours Valley Mills is scheduled.
   y = number of hours Marlin is scheduled.
   z = number of hours Hillsboro is scheduled.

   $10x + 7y + 5z = 1365$
   $12x + 10y + 4z = 1530$
   $6x + 8y + 13z = 1890$

   Solution:
   Valley Mills scheduled for 60 hours
   Marlin scheduled for 45 hours
   Hillsboro scheduled for 90 hours

7. x = the number of carton A
   y = the number of carton B
   z = the number of carton C

   $2x + 6y + 4z = 50$
   $5x + 8y + 6z = 78$
   $3x + 2y + 10z = 52$

   Solution:
   4 of carton A
   5 of carton B
   3 of carton C

8. x = number of plain hamburgers
   y = number of double cheeseburgers
   z = number of regular cheeseburgers

   $x + y + z = 86$
   $x + 2y + z = 100$
   $4y + 2z = 140$

   Solution:
   30 plain hamburgers
   14 double cheeseburgers
   42 regular cheeseburgers

9. x = number of one-bedroom units
   y = number of two-bedroom units
   z = number of three-bedroom units

   $x + y + z = 225$
   $y + z = 2x$
   $x = 3z$

   Solution:
   x=75
   y = 125
   z = 25

10. x = number of children at the show
    y = number of students at the show
    z = number of adults at the show

    $x + y + z = 900$
    $2x + 3y + 4z = 2800$
    $2z = x + y$

    Solution:
    x=200
    y=400
    z=300

11. x = number of barrels of mix A
    y = number of barrels of mix B
    z = number of barrels of mix C
    w = number of barrels of mix D

    $30x + 30y + 30z + 60w = 900$
    $50z + 75y + 25z + 25w = 750$
    $30x + 20y + 20z + 50w = 700$

Initial matrix final matrix

$$\left[\begin{array}{cccc|c} 30 & 30 & 30 & 60 & 900 \\ 50 & 75 & 25 & 25 & 750 \\ 30 & 20 & 20 & 50 & 700 \end{array}\right] \overset{\text{rref}}{\rightarrow} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 10 \\ 0 & 1 & 0 & -1 & -5 \\ 0 & 0 & 1 & 2 & 25 \end{array}\right]$$

Parametric Solution:
$x = 10 - w$
$y = -5 + w$
$z = 25 - 2w$
$w = any\ number$

Now place restrictions on the parameter. Since we are not told a maximum number of barrels that can be bought assume there is no limit. We know the number of barrels bought has to be non-negative.

$x \geq 0$	$y \geq 0$	$z \geq 0$	$w \geq 0$
$10 - w \geq 0$	$-5 + w \geq 0$	$25 - 2w \geq 0$	
$10 \geq w$	$w \geq 5$	$25 \geq 2w$	
$w \leq 10$		$12.5 \geq w$	
		$w \leq 12.5$	

The restriction on the parameter is that w must be an integer and $5 \leq w \leq 10$ (i.e. $w = 5, 6, 7, 8, 9, 10$)

12. x = number of cars purchased with 6,000 gallon capacity
y = number of cars purchased with 8,000 gallon capacity
z = number of cars purchased with 18,000 gallon capacity

$x + y + z = 24$
$6000x + 8000y + 18000z = 250000$

Initial matrix and final matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 24 \\ 6000 & 8000 & 18000 & 250000 \end{array}\right] \overset{\text{rref}}{\rightarrow} \left[\begin{array}{ccc|c} 1 & 0 & -5 & -29 \\ 0 & 1 & 6 & 53 \end{array}\right]$$

Parametric solution:
$x = -29 + 5z$
$y = 53 - 6z$
$z = any\ number$

We can not buy a part of a tank car. So z must be an integer. We also know that all of the variables must be greater than or equal to zero.

$x \geq 0$	$y \geq 0$	$z \geq 0$
$-29 + 5z \geq 0$	$53 - 6z \geq 0$	
$5z \geq 29$	$53 \geq 6z$	
$z \geq 5.8$	$53/6 \geq z$	
	$z \leq \frac{53}{6} \approx 8.8333$	

In addition we know that the variables can not be any larger than 24

$x \leq 24$	$y \leq 24$	$z \leq 24$
$-29 + 5z \leq 24$	$53 - 6z \leq 24$	
$5z \leq 53$	$29 \leq 6z$	
$z \leq 10.6$	$29/6 \leq z$	
	$z \geq \frac{29}{6} \approx 4.8333$	

Taken all together, we find that $z = 6$, 7, 8. This problem ends up having only three solutions.

13. x = the number of chihuahuas bought
y = the number of cats bought
z = the number of dogs bought

$x + y + z = 14$
$4x + 7y + 16z = 116$

Solution:
$x = -6 + 3z$
$y = 20 - 4z$
$z = any\ number$

We can not buy a part of a pet. So z must be an integer. We also know that all of the variables must be greater than or equal to zero.

$x \geq 0$	$y \geq 0$	$z \geq 0$
$-6 + 3z \geq 0$	$20 - 4z \geq 0$	
$3z \geq 6$	$20 \geq 4z$	
$z \geq 2$	$5 \geq z$	

In addition we know that the variables can not be any larger than 14

$x \leq 14$	$y \leq 14$	$z \leq 14$
$-6 + 3z \leq 14$	$20 - 4z \leq 14$	
$3z \leq 20$	$6 \leq 4z$	
$z \leq \frac{20}{3} \approx 6.6667$	$1.5 \leq z$	

Taken all together, we find that $z = 2, 3, 4$, or 5

14. s = the number of small shirts at the end of the day.
m = the number of medium shirts at the end of the day.
l = the number of large shirt at the end of the day.

$s + m + l = 45$
$8s + 10m + 13l = 480$
$8(2s) + 10(4m) + 13(5l) = 1940$

Solution:
15 small shirts
10 medium shirts
20 large shirts

15. x = number of tank cars purchased with 7,000 gallon capacity
y = number of tank cars purchased with 9,000 gallon capacity
z = number of tank cars purchased with 20,000 gallon capacity

$x + y + z = 40$
$7000x + 9000y + 20000z = 400000$

Solution:
$x = -20 + 5.5z$
$y = 60 - 6.5z$
$z = 4, 6,$ or 8

16. x = number of wood pens made
y = number of silver pens made
z = number of gold pens made

$x + .5y + 3z = 12000$
$2x + 3y + 2z = 9600$

Solution:
$x = 15600 - 4z$
$y = -7200 + 2z$
$3600 \le z \le 3900$ and z is an integer

17. x = the number of evil sorcerers slain.
y = the number of warriors slain.
z = the number of orcs slain.

$x + y + z = 370$
$2x + 4y + z = 560$
$y = 6x$

Solution:
10 evil sorcerers
60 warriors
300 orcs

18. x = the amount of money invested in the QX company
y = the amount of money invested in the RY company
z = the amount of money invested in the KZ company

$x + y + z = 17300$
$2z = y$
$1.5\left(\frac{x}{130}\right) + 1\left(\frac{y}{75}\right) + 2\left(\frac{z}{90}\right) = 251$

Solution:
\$6,500 invested in QX
\$7,200 invested in RY
\$3,600 invested in KZ

19. x = number of 12-ounce(small) cups sold
y = number of 16-ounce(medium) cups sold
z = number of 20-ounce(large) cups sold

$x + y + z = 23$
$12x + 16y + 20z = 376$
$x + 2y + 3z = 48$

Solution:
$x = z - 2$
$y = -2z + 25$
$z = 2, 3, 4, \cdots 12$

20. (a) the variables x, y, z, and w are the average number of vehicles on that section of the road.
Note: the number of vehicles entering the intersection must equal the number of vehicles exiting the intersection.
$x + y = 1400$
$y + z = 1200$
$z + w = 1100$
$x + w = 1300$

Solution:
$x = 1300 - w$
$y = 100 + w$
$z = 1100 - w$
w =any number

(b) $200 \le w \le 1000$

(c) To get this restriction we need $y = 150$. This means that $w = 50$. Since this is outside of the restrictions found in part b, the answer is no.

21. (a) $x + y = 1300$
$y + z = 1300$
$z + w - m = 800$
$v - m = 500$
$x + w - v = 300$

Solution:
columns in the matrix are x,y,z,v,m,w
$x = 800 + m - w$
$y = 500 - m + w$
$z = 800 + m - w$
$v = 500 + m$
m = any number
w = any number

(b) The only restriction that we can place on the parameters m and w is that they have to be non-negative, i.e. ≥ 0. We can not say anything else since the choice of m will affect the possible choices of w.

22. (a) $\begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -4 \\ 0 & 4 \\ 8 & -2 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ -1 & -2 \\ -5 & 3 \end{bmatrix}$

(b) Not possible, the dimension don't match.

(c) $\begin{bmatrix} 7 & -14 & 0 \\ -7 & 21 & 14 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 8 \\ -4 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 9 & -14 & 8 \\ -11 & 25 & 12 \end{bmatrix}$

(d) Not possible, the dimensions don't match.

(e) $\begin{bmatrix} 3 & -6 & 0 \\ -3 & 9 & 6 \end{bmatrix} - \begin{bmatrix} 2 & -2 & 6 \\ 0 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -4 & -6 \\ -3 & 5 & 4 \end{bmatrix}$

23. (a) $\begin{bmatrix} 5a & -15 & 5 \\ 0 & 5 & 20 \end{bmatrix} + \begin{bmatrix} 6a & -6 & 2 \\ 0 & 2 & 6 \end{bmatrix} = \begin{bmatrix} 11a & -21 & 7 \\ 0 & 7 & 26 \end{bmatrix}$

(b) Not possible, the dimensions don't match.

(c) Not possible, the dimensions don't match.

(d) $\begin{bmatrix} 6 & 0 \\ -6 & 9 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 6+a & b \\ -6+c & 9+d \end{bmatrix}$

(e) $\begin{bmatrix} 2 & 6 & 2j \\ -4 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 3 & -5 \\ 7 & k & 2 \end{bmatrix} - C^T =$
$\begin{bmatrix} 2 & 9 & 2j-5 \\ 3 & k & 6 \end{bmatrix} - \begin{bmatrix} 2 & 7 & 1 & b \\ -1 & 0 & -3 & \end{bmatrix} =$
$\begin{bmatrix} 0 & 8 & 2j-5-b \\ 4 & \text{k} & 9 \end{bmatrix}$

24. (a) $\begin{bmatrix} 6x+2 & 12+2u \\ -3-2z & 14 \end{bmatrix} = \begin{bmatrix} 2y & 5 \\ 7 & y \end{bmatrix}$
Now solve these equations:
$6x + 2 = 2y$
$12 + 2u = 5$
$-3 - 2z = 7$
$14 = y$
Answer:
$y = 14$, $z = -5$, $u = -3.5$, and $x = 26/6$

(b) $\begin{bmatrix} 3 & -4x+5y \\ 6y+10x & 12 \end{bmatrix} = \begin{bmatrix} 3 & 22 \\ -3 & 12 \end{bmatrix}$

Now solve these equations:
$-4x+5y=22$
$6y+10x=-3$
Answer: $x=\frac{-147}{74}$ and $y=\frac{104}{37}$

(c) $\begin{bmatrix} 2 & 3x \\ 6x & 6 \end{bmatrix} + 2\begin{bmatrix} 3 & -2y \\ 6y & -1 \end{bmatrix} = \begin{bmatrix} 8 & -13 \\ 84 & 4 \end{bmatrix}$

$\begin{bmatrix} 8 & 3x-4y \\ 6x+12y & 4 \end{bmatrix} = \begin{bmatrix} 8 & -13 \\ 84 & 4 \end{bmatrix}$

Now solve these equations:
$3x-4y=-13$
$6x+12y=84$
Answer: $x=3$ and $y=5.5$

(d) $\begin{bmatrix} x-6y & 2-2z \\ y-14 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 2x & -1 \end{bmatrix}$

Now solve these equations:
$x-6y=4$
$2-2z=0$
$y-14=2x$
Answer: $x=-8$, $y=-2$, and $z=1$

25. (a) 3×1
(b) 5×3
(c) Not possible.
(d) 4×4
(e) Not possible.
(f) 3×5

26. (a) False. Try with the folowing.
$A=\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
(b) True
(c) False, it is a 4×4 matrix.

27. (a) Not possible
(b) $\begin{bmatrix} 13 & -7 \\ 4 & 3 \end{bmatrix}$
(c) Not possible
(d) $\begin{bmatrix} 2 & -2 & 4 \\ -2 & 7 & 6 \end{bmatrix}$
(e) $\begin{bmatrix} 17 & 20 \\ 7 & 6 \end{bmatrix}$
(f) $\begin{bmatrix} a-2c & b-2d \\ 2c & 2d \\ 4a-c & 4b-d \end{bmatrix}$
(g) $\begin{bmatrix} a & -a+2b & 3a+b \\ c & -c+2d & 3c+d \end{bmatrix}$

28. $\begin{bmatrix} 2 & 6 & -4 \\ 10 & 16 & -8 \\ -12 & 20 & 10 \end{bmatrix}$

29. $AB=\begin{bmatrix} x+5 & 2 \\ y+5 & 2 \end{bmatrix}$ and $BA=\begin{bmatrix} x & 1 \\ 5x+2y & 7 \end{bmatrix}$

30. Note: either multiple the entire matrix or only use the row and column needed for the answer.
(a) $C_{1,3}=0+9+20=29$
(b) $D_{3,1}=0+0+50+4=54$

31. $\begin{bmatrix} -5 & 30 \\ 6+x-2y & 15+5y \end{bmatrix} = \begin{bmatrix} -5 & y+2z \\ 1 & 35 \end{bmatrix}$

Now solve these equations:
$30=y+2z$
$6+x-2y=1$
$15+5y=35$
Answer: $x=3$, $y=4$, and $z=13$

32. (a) $BM=\begin{bmatrix} 2910 & 8970 \end{bmatrix}$
There is no meaning for these numbers since the lables of the rows/collumns do not match up.

(b) $ML^T=\begin{bmatrix} 7200 \\ 2700 \end{bmatrix}$
The 7200 is the amount of vitamin A and the 2700 is the amount of vitamin C that is consumed at lunch.

(c) $(B+L)M=\begin{bmatrix} 6840 & 21480 \end{bmatrix}$
There is no meaning for these numbers since the lables of the rows/collumns do not match up.

(d) $M(BL)^T=\begin{bmatrix} 11200 \\ 4040 \end{bmatrix}$
The 11200 is the amount of vitamin A and the 4040 is the amount of vitamin C that is consumed together at breakfast and lunch.

33. (a) $x=3-4y+w$
$z=4-2w$
$y, w=$ any number
(b) $x=8$, $y=2$, and $z=5$
(c) No solution.
Note: no solution mean no solution for ALL of the variables. Do not say that x=8, z=5, no solution.
(d) $x=3+z$
$y=1-2z$
$z=$ any number
(e) $x=0$, $y=4$, and $z=2$
(f) No solution
(g) $x=-2$, $y=1$, $z=3$
(h) No solution

34. (a) $\left[\begin{array}{ccc|c} 1 & 0 & 1 & -12 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 7 & -7 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 2 & 19 & 27 \\ 0 & 2 & -12 & -4 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & -2 & 4 & 4 \\ 0 & 18 & 3 & 10 \\ 3 & 0 & 6 & 1 \end{array}\right]$

(d) $\left[\begin{array}{ccc|c} 4 & 0 & 26 & -7 \\ 0 & 4 & -2 & 5 \\ 0 & 7 & 74 & 35 \\ 5 & 1 & 2 & 5 \end{array}\right]$

35. (a) i. $x = 2 - 2z$
$y = 2 + z$
$z =$ any number

ii. no solution

(b) i. $x = 12$, $y = -22$, and $z = 41$

ii. $x = \frac{-2}{17}$, $y = \frac{-10}{17}$, and $z = \frac{-60}{17}$

(c) no solution

(d) $x = 7 - 10z$
$y = 1 + 3z$
z = any number

(e) $x = 8$, $y = 3$, and $z = 5$

(f) $x = \frac{70}{3} + z$
$y = \frac{110}{3} - 2z$
$z =$any number

(g) $x = 1$, $y = -2$, $z = 3$, and $w = 8$

(h) $x = 26 - 2y + 14w$
$z = 7 + 6w$
y = any number
w = any number