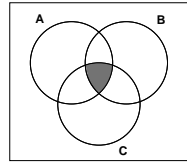


## Chapter 6 Homework Solutions

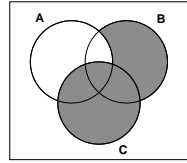
Compiled by Joe Kahlig

- $\{0, 1, 11, 13\}$
  - $B \cap A = \{11\}$   
Answer:  $\{0, 1, 11, 13\}$
  - $\{1, 13\}$
  - False  
It is true that  $\emptyset \subset A$ , but it is not an element of  $A$ .
  - False  
 $C$  contains 0 where as  $A$  does not.
- $A \cup B = \{0, 2, 3, 4, 5, 6, 7, 8\}$   
 $(A \cup B)^C = \{1, 9\}$   
Answer:  $\{1\}$
  - $\{2\}$
  - $C^C = \{1, 3, 4, 5, 6, 7, 8\}$   
 $B \cap C^C = \{3, 4, 5, 6, 7, 8\}$   
Answer:  $\{0, 2, 3, 4, 5, 6, 7, 8\}$
  - $D \cap A = \{2, 4, 8\}$   
 $(D \cap A)^C = \{0, 1, 3, 5, 6, 7, 9\}$   
Answer:  $\{3, 5, 6, 7\}$
  - $\{ \}$  or  $\emptyset$
  - $A \cup D = \{0, 1, 2, 3, 4, 5, 6, 8\}$   
Answer:  $\{7, 9\}$
  - $D^C = \{0, 6, 7, 9\}$   
 $B \cap D^C = \{6, 7\}$   
 $(B \cap D^C)^C = \{0, 1, 2, 3, 4, 5, 8, 9\}$   
 $A^C = \{1, 3, 5, 7, 9\}$   
Answer:  $\{1, 3, 5, 9\}$
  - $D^C = \{0, 6, 7, 9\}$   
Answer:  $\{6, 7\}$
  - $U$  or  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
  - Yes
  - No, it is true that  $\emptyset \subset A$ , but it is not an element of  $A$ .
  - No, 4 is a number and not a set.
  - $2^5 = 32$
  - $2^6 - 1 = 63$
  - Yes
  - No
- $\emptyset, \{m\}, \{n\}, \{p\}, \{m, n\}, \{m, p\}, \{n, p\}, \{m, n, p\}$
- $\emptyset, \{m\}, \{n\}, \{p\}, \{m, n\}, \{m, p\}, \{n, p\}$
- Remember one method is to label the regions of the venn diagram with letters and then shade the regions that are given in the answer to the computation of the set operation.

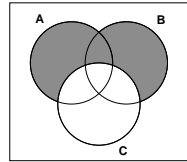
(a)  $A \cap B \cap C$



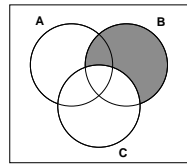
(b)  $(A^C \cap B) \cup C$



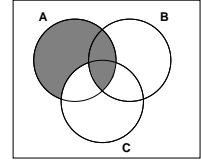
(c)  $(A \cup B) \cap C^C$



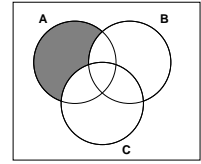
(d)  $B \cap C^C$



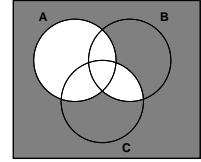
(e)  $A \cap (B \cup C^C)$



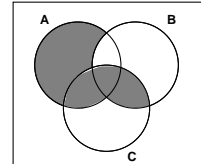
(f)  $A \cap B^C \cap C^C$



(g)  $A^C \cap (B^C \cup C^C)$



(h)  $(A \cap B^C) \cup (B \cap C)$



6. (a)  $(A \cup M \cup E)^C$

(b)  $(E \cup M) \cap A^C$

(c) Those students that have not had a course in Economics but have had a course in Accounting.

(d) The students that have had a Marketing class but not an Accounting class combined with the students that have had an Economics class.

7. (a)  $(D \cup C) \cap F^C$

(b)  $D \cap C^C \cap F^C$  or  $D \cap (C \cup F)^C$

8. (a)  $2^{14} = 16384$

$$(b) \begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ 18 &= 14 + 10 - n(A \cap B) \\ n(A \cap B) &= 6 \end{aligned}$$

(c)  $A \cap B^C$  is how many things are in A but not in B. Since A has 14 items and A and B have an overlap of 6 items then there are 8 items in A but not in B. Or use a venn diagram.

Answer: 8

9.  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ .

$$n(A \cup B) = 15 + 12 - 7$$

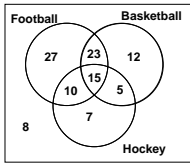
$$n(A \cup B) = 20$$

10.  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ .

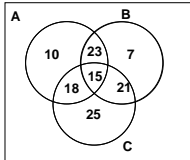
$$25 = 15 + n(B) - 7$$

$$n(B) = 17$$

## 11. Venn diagram



## 12. It is not necessary to make a venn diagram, but it helps.



- (a) 119  
 (b)  $23 + 21 + 18 = 62$   
 (c) 23  
 (d)  $25 + 21 = 46$
13. (a)  $20 + 15 + 10 + 8 + 12 + 9 = 74$   
 (b) 10  
 (c)  $10 + 15 + 9 + 12 + 20 + 45 + 60 = 171$   
 (d)  $15 + 12 = 27$
14. (a)  $20 + 12 + 45 = 77$   
 (b)  $15 + 12 + 8 + 9 + 10 + 45 = 99$   
 (c)  $15 + 9 + 12 = 36$   
 (d)  $60 + 45 = 105$
15. (a)  $30 = 8 + 10 + 12$   
 (b)  $62 = 8 + 12 + 8 + 12 + 12 + 10$   
 (c)  $28 = 8 + 12 + 8$
16.  $5 * 4 * 3 * 2$
17.  $26 * 25 * 24 * 5 * 5$
18. The first digit must be 6, 7, 8, or 9. The second and third digit are free to be anything but the last digit must be odd.  
 Answer:  $4 * 10 * 10 * 5$
19. The first digit is 3 or bigger and the last digit is either 0 or 5.  
 Answer:  $7 * 10 * 10 * 10 * 2$
20. 2 choices for 6 true/false and 4 choices for 5 multiple choice.  
 Answer:  $2 * 2 * 2 * 2 * 2 * 2 * 4 * 4 * 4 * 4 * 4 = 2^6 * 4^5$
21. The first 9 questions have 5 options (4 choices and one blank) and for the last 6 questions have 7 choices (6 choices and one blank).  
 Answer:  $5^9 * 7^6$

22.  $3 * 14 * 13$

23. (a) Since the number of boys and girls are equal then the only way that they can alternate is to start with a boy.

$$\frac{5}{\text{Boy}} * \frac{4}{\text{Girl}} * \frac{4}{\text{Boy}} * \frac{3}{\text{Girl}} * \frac{3}{\text{Boy}} * \frac{2}{\text{Girl}} * \frac{2}{\text{Boy}} * \frac{1}{\text{Girl}} * \frac{1}{\text{Boy}}$$

Answer:  $5! * 4!$ 

- (b) First place the girls at the start of the row and the boys at the end.

$$\frac{4}{\text{Girl}} * \frac{3}{\text{Girl}} * \frac{2}{\text{Girl}} * \frac{1}{\text{Girl}} * \frac{5}{\text{Boy}} * \frac{4}{\text{Boy}} * \frac{3}{\text{Boy}} * \frac{2}{\text{Boy}} * \frac{1}{\text{Boy}}$$

Now determine how many ways the girls can be moved down the row with them still sitting next to each other( 6 ways).

Answer:  $(4! * 5!) * 6$ 

24. Too many boys, not enough girls.

Answer: 0

25. First consider the row with Jacob or Fred in the middle seat (2 options) and then Randy and Susan in the first two seats(2 \* 1 options).

$$\frac{2}{\text{R/S}} * \frac{1}{\text{R/S}} * \frac{4}{\text{Any}} * \frac{2}{\text{J/F}} * \frac{3}{\text{Any}} * \frac{2}{\text{Any}} * \frac{1}{\text{Any}}$$

Now determine how many places Randy and Susan can be placed in the row (4 ways).

Answer:  $4 * (2 * 1 * 4 * 2 * 3 * 2 * 1) = 384$ 

26. Note the license plate has a specified form.

$$\frac{26}{\text{Letter}} * \frac{25}{\text{Letter}} * \frac{9}{1-9} * \frac{9}{1-9} * \frac{2}{\text{Symbol}}$$

27. This is the same as the previous question except the letters and numbers positions can be shuffled. You need to determine how many ways that this can happen.

Answer:  $(26 * 25 * 9 * 9 * 2) * 6$ 

28. There should be only one blank for the code for the month since 01, ... 12 are each a fixed item. If you do two blanks, then you allow things like 13, 14, ..., 19.

$$\frac{3}{\text{J,H,T}} * \frac{12}{01-12} * \frac{10}{0-9} * \frac{10}{0-9} * \frac{10}{0-9} * \frac{10}{0-9}$$

29. First consider the case when all three letters are on the left and the digits are on the right.

$$\frac{26}{\text{Letter}} * \frac{25}{\text{Letter}} * \frac{24}{\text{Letter}} * \frac{10}{0-9} * \frac{9}{0-9}$$

Now determine how many ways you can rearrange the letter/number positions.

Answer:  $(26 * 25 * 24 * 10 * 9) * 10$

30.  $4!$  will order the blue books,  $5!$  will order the green books, and  $2!$  will order the red books. Now determine how many ways the three different colors can be placed in a row.

Answer:  $(4! * 5! * 2!) * 3!$

31. Similar to the problem 30. Uses author names instead of colors.

Answer:  $(3! * 2! * 6! * 5!) * 4!$

32. The problem asks for at least one 5 being rolled.

**Method 1:** count all the cases.

Exactly one five:  $1 * 5 * 5 + 5 * 1 * 5 + 5 * 5 * 1$

Exactly two fives:  $1 * 1 * 5 + 1 * 5 * 1 + 5 * 1 * 1$

Exactly three fives:  $1 * 1 * 1$

Now add these results to get the final answer.

**Method 2:** You don't want an outcome with no fives.

Total:  $6 * 6 * 6$

Don't want:  $5 * 5 * 5$

**Answer:**  $6^3 - 5^3 = 91$

33. This problem is similar as problem 32. Use the second method.

Answer:  $6^5 - 5^5 = 4651$

34. There are 6 even numbers and 7 odd numbers in this range. If the first two people pick an even number and the last two pick an odd number then there are  $6 * 6 * 7 * 7$  possibilities. Now shuffle which two people are going to pick the even numbers.

Answer:  $(6 * 6 * 7 * 7) * 6 = 10,584$

35. Use the total - don't want method. There are a total of  $8^4$  codes. The number of codes without a vowel is  $5^4$ .

Answer:  $8^4 - 5^4 = 3471$

36. First consider the first two people to have the same birthday and the last two have different birthdays. There are  $365 * 1 * 364 * 363$  ways for this to happen. Now shuffle which two people have the same birthday.

Answer:  $6(365 * 1 * 364 * 363)$

37. Since the number of boys and girls are not the same and we have an odd number of people in the row, we must compute both forms of alternating and add the results.

$$\frac{6}{B} * \frac{9}{G} * \frac{5}{B} * \frac{8}{G} * \frac{4}{B} + \frac{9}{G} * \frac{6}{B} * \frac{8}{G} * \frac{5}{B} * \frac{7}{G}$$

38. Method 1: At least one digit is a seven.

$$1 * 1 * 1 + 8 * 1 * 1 + 1 * 9 * 1 + 1 * 1 * 9 + 8 * 9 * 1 + 8 * 1 * 9 + 1 * 9 * 9$$

Method 2: Total - Don't want

$$9 * 10 * 10 - 8 * 9 * 9$$

39. (a) 108336

(b) 108336

(c) 12600

(d) 1200

(e) 3744

(f) 0.97959

(g) 0.012707

(h) 450

(i) 450

(j) 1

(k) 1

(l) 14400

40. Order is important since the secretaries are each working for a different manager.

Answer:  $210 = P(7, 3)$

41. Order is not important.

$$\frac{C(13,2)}{\text{Heart}} * \frac{C(13,3)}{\text{Spades}} * \frac{C(26,1)}{\text{Other}}$$

42. Order is not important.

$$\frac{C(5,3)}{\text{Fr.}} * \frac{C(4,2)}{\text{Soph.}} * \frac{C(2,1)}{\text{Other}}$$

43. since this problems has repetition of identical objects, this is a distinct rearrangement problem.

Answer:  $\frac{11!}{2!3!6!}$  or  $C(11, 2) * C(9, 3) * C(6, 6)$

44. Since all of the positions have the same title, order is not important.

$$C(10, 2) * C(8, 3)$$

45. Method 1: There are 10 symbols(4 zeroes and 6 ones) so the total number of ways to arrange these is  $10!$ , but since some symbols are repeated we need to divide by the repeats.

Answer:  $\frac{10!}{4!6!}$

Method 2: There are 10 blanks to fill and we need 4 of them to be zeroes. The rest of the positions will be ones.

Answer:  $C(10, 4) * C(6, 6) = C(10, 4)$  or  $C(10, 6) * C(4, 4) = C(10, 6)$

46. Method 1: If there were no restrictions there would be  $7^3$  possibilities. There are only 7 possibilities for one person to win all of the awards, so the answer is the difference.

Answer:  $7^3 - 7 = 336$

Method 2: First count the distinct wins of which there are  $7 * 6 * 5$  possibilities. Then choose two of the three awards to be given to one person and count the number of ways to give two awards to 7 people. Add the results.

Answer:  $7 * 6 * 5 + C(3, 2) * 7 * 6.$

47. Since the scholarships are different, order is important.

$$\text{Answer: } P(15, 4) = 15 * 14 * 13 * 12$$

48. (a) order is not important.

$$\frac{C(3,3)}{J/M/S} * \frac{C(9,3)}{\text{other}}$$

(b) order is not important.

$$\frac{C(3,1)}{J/M/S} * \frac{C(9,5)}{\text{other}}$$

49.  $\frac{10!}{2!2!3!}$

50. Note: the capital M and the lowercase m are different objects.

$$\frac{11!}{3!2!}$$

51. (a) Exactly two the same color has the following cases: exactly two red and one other; exactly two green and one other; and exactly two black and one other.

Answer:

$$C(2, 2) * C(12, 1) + C(5, 2) * C(9, 1) + C(7, 2) * C(7, 1)$$

(b) at least two green means exactly 2 green or exactly 3 green.

$$\text{Answer: } C(5, 2) * C(9, 1) + C(5, 3)$$

52. (a) Choose 4 of the 10 defective transistors and 3 of the 50 good transistors.

$$\text{Answer: } C(10, 4) * C(50, 3)$$

(b) Total - don't want (i.e. All defective).

$$\text{Answer: } C(60, 7) - C(10, 7) * C(50, 0)$$

53. (a) Consider the cases shown in the table.

red	purple	green	red	purple	green
3	1	2	4	1	1
3	2	1	4	2	0
3	3	0	5	1	0

Answer:

$$C(5, 3)C(4, 1)C(6, 2) + C(5, 3)C(4, 2)C(6, 1) + C(5, 3)C(4, 3) + C(5, 4)C(4, 1)C(6, 1) + C(5, 4)C(4, 2) + C(5, 5)C(4, 1)$$

(b) Use the union formula:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

where A is exactly 2 red and B is exactly 3 purple. The overlap  $A \cap B$  is the case with 2 red, 3 purple and 1 green.

Answer:

$$C(5, 2)C(10, 4) + C(4, 3)C(11, 3) - C(5, 2)C(4, 3)C(6, 1)$$

54. (a) Note: No freshmen can attend (there are only two).

$$\text{Answer: } C(5, 3) + C(12, 3)$$

(b) Consider the cases shown in the table.

fr	jr	soph
1	0	2
2	0	1
2	1	0

Answer:

$$C(2, 1) * C(5, 2) + C(2, 2) * C(5, 1) + C(2, 2) * C(12, 1)$$

55. Order is not important since we are just selecting questions to answer. not figuring out the order the questions are answered. Choose 2 of the first 3 and 8 of the remaining 12.

$$\text{Answer: } C(3, 2) * C(12, 8)$$

56.  $C(3, 2) + C(5, 2) + C(4, 2) = 19$

57. (a)  $C(30, 10)$

(b) order is not important.

$$\frac{C(8, 3)}{15 \text{ yr old}} * \frac{C(22, 7)}{\text{other}}$$

58. order is not important

$$\frac{C(7, 3)}{\text{red}} * \frac{C(5, 2)}{\text{black}} * \frac{C(9, 2)}{\text{Other}}$$

59. (a)  $C(7, 4) + C(4, 4) + C(5, 4)$

(b) Note: no way to get 3 purple so this case is not used.

$$\frac{C(7, 3)C(11, 1)}{3 \text{ red } 1 \text{ other}} + \frac{C(4, 3)C(14, 1)}{3 \text{ green } 1 \text{ other}} + \frac{C(5, 3)C(13, 1)}{3 \text{ black } 1 \text{ other}}$$

60. (a) picking 7 items so you need one more fruit that is not an apple nor a peach, i.e. 1 other.

$$\text{Answer: } C(30, 2)C(14, 4)C(21, 1)$$

(b) Use the union formula:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

where A is exactly 4 plums and B is exactly 3 pears. The overlap  $A \cap B$  is the case with 4 plums and 3 pears.

Answer:

$$C(6, 4)C(59, 3) + C(15, 3)C(50, 4) - C(6, 4)C(15, 3)$$

61. Choose 3 of Billy's 43 cards and 3 of Scottie's 36 cards.

$$\text{Answer: } C(43, 3)C(36, 3)$$

62.  $C(7, 2)C(15, 2) + C(7, 3)C(15, 1) + C(7, 4)$

63. Distinct rearrangement problem.

$$\frac{20!}{7!2!4!6!} \text{ or } C(20, 7)C(13, 2)C(11, 4)C(7, 6)C(1, 1)$$

64. Use the union formula:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

where A is exactly 2 hearts and B is exactly 3 spades. The overlap  $A \cap B$  is the case with 2 hearts, 3 spades and 1 other card.

Answer:

$$C(13, 2) * C(39, 4) + C(13, 3) * C(39, 3) - C(13, 2) * C(13, 3) * C(26, 1)$$

65. Method 1: since there are 10 blocks and 15 positions let e represent the empty spots. Then

rrrrrrggggeeeee

would represent how the blocks could be placed on the sheets of papers. To get another arrangement, then just shuffle the letters.

Method 2: pick the positions that the blocks will be placed. Since each block will be on a different sheet of paper, we do not have repeats. Also order is not important. Thus  $C(15, 6)$  would place the red blocks into their positions. do something similar for the green blocks.

Answer: 630630

66.  $C(5, 3)C(6, 4)C(4, 3) + C(5, 3)C(6, 5)C(4, 2) + C(5, 3)C(6, 6)C(4, 1)$