Week in Review–Additional Chapter 2 Material

1. (a) no since there is a 3 in row 1 column 3 position.
   (b) yes
   (c) yes
   (d) yes

2. (a) no solution
   (b) $x = 9$, $y = 10$, and $z = 6$
   (c) $x = 2 - 4z$
   $y = 9 - 5z$
   $z =$ any number
   (d) $x = 7 - 2y - 2w$
   $z = 3 - 4w$
   $y =$ any number
   $w =$any number
   (e) $x = 4$, $y = 2$, and $z = 8$

3. See the streaming video for the work.
   (a) $x = -6$, $y = 12$, $z = 4$
   (b) $x = 3$, $y = -2$, $z = 1$

4. (a) first rewrite the equations as shown.
   $3x + y = 9$
   $x - y + z = 4$
   $3x + z = 11$
   $4x - y + 2z = 15$

$$\left[\begin{array}{cccc} 3 & 1 & 0 & 9 \\ 1 & -1 & 1 & 4 \\ 3 & 0 & 1 & 11 \\ 4 & -1 & 2 & 15 \end{array}\right] \overset{\text{rref}}{\longrightarrow} \left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

   Answer: $x = 2$, $y = 3$, and $z = 5$

   (b)
$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 10 \\ 2 & 7 & -1 & 21 \\ 4 & 13 & 1 & 41 \end{array}\right] \overset{\text{rref}}{\longrightarrow} \left[\begin{array}{ccc|c} 1 & 0 & 10 & 7 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

   Answer:
   $x = 7 - 10z$
   $y = 1 + 3z$
   $z =$ any number.

   note: no restrictions can be placed on the parameter since this was not a word problem.

   (c)
$$\left[\begin{array}{ccc|c} 3 & 2 & 5 & 7 \\ 1 & 4 & 1 & 13 \\ 4 & -5 & 2 & -9 \\ 5 & 10 & 7 & 32 \end{array}\right] \overset{\text{rref}}{\longrightarrow} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

   Answer: no solution.

5. (a) Set up of the problem:
   $x =$ the number of old dvds bought
   $y =$the number of semi-new dvds bought
   $z =$ the number of new dvds bought.

   $x + y + z = 60$
   $10x + 16y + 22z = 840$

   Solution:
   $x = 20 + z$
   $y = 40 - 2z$
   $z =$any number

   Now place restrictions on the parameter z. This is the mathematical process. You could also do this by inspecting the parametric solution for what values of z will make sense.
   We know that the number of dvds bought must be greater than or equal to zero.

$$\begin{array}{ccc} x \geq 0 & y \geq 0 & z \geq 0 \\ 20 + z \geq 0 & 40 - 2z \geq 0 & \\ z \geq -20 & 40 \geq 2z & \\ & 20 \geq z & \end{array}$$

   We also know that the number of dvds bought must be less than 60.

$$\begin{array}{ccc} x \leq 60 & y \leq 60 & z \leq 60 \\ 20 + z \leq 60 & 40 - 2z \leq 60 & \\ z \leq 40 & -2z \leq 20 & \\ & z \geq -10 & \end{array}$$

   Thus we get that $0 \leq z \leq 20$ and z must be an integer or in other words $z = 0, 1, 2, 3, ..., 20$

   (b) 21 different solutions.