Math 152 Week in Review: Sections 11.5, 11.6

Solutions and questions can be found at the link:
https://www.math.tamu.edu/~kahlig/152WIR.html

The Alternating Series Test (AST): If the alternating series

$\sum_{n=1}^{\infty} (-1)^{(n-1)} b_n$ with $b_n > 0$ satisfies: (1) $b_{n+1} \leq b_n$ for all n and (2) $\lim_{n \to \infty} b_n = 0$

then the series is convergent.

Alternating Series Estimation Theorem: If $s = \sum_{n=1}^{\infty} (-1)^{(n-1)} b_n$ is the sum of an alternating series that satisfies:

(a) $0 < b_{n+1} \leq b_n$ and (b) $\lim_{n \to \infty} b_n = 0$

then $|R_n| = |s - s_n| \leq b_{n+1}$

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1. Determine if the series converges or diverges.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (n+5)}{n^2 + 3n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{2n+1} \cos\left(\frac{\pi}{n}\right)$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\left(\frac{-1}{3}\right)^n}{n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin\left((n+\frac{1}{2})\pi\right)}{1+\sqrt{n}}$

2. Use the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{(n+3)!}$ and the fact it converges by the alternating series test for the following.

   (a) Estimate the sum of the series by s_5

   (b) Find an upper bound for the error in the estimate.

   (c) Is the estimate, s_5, more or less than the actual sum?

3. How many terms of the series do we need to add in order to find the sum so that the $|\text{error}| < 0.0005$?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n3^n}$$

Definition: A series $\sum a_n$ is called **absolutely convergent** if the series $\sum |a_n|$ is convergent.

If a series $\sum a_n$ is absolutely convergent then it is also convergent.

Definition: A series $\sum a_n$ is called **conditionally convergent** if the series $\sum |a_n|$ is divergent and the series $\sum a_n$ is convergent.

4. Determine if the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{\cos n}{n^4}$$

5. Determine if the series is absolutely convergent, conditionally convergent, or divergent.
$\sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{\sqrt{n}}$

The Ratio Test:

(a) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L$, with $0 \le L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is **absolutely convergent** (and therefore convergent).

(b) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

Note: If the limit for the ratio test is 1, then this test fails to give any information. Try something else.

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6. For which series is the Ratio Test inconclusive? (fails to give a definite answer)

(a) $\sum_{n=1}^{\infty} \frac{2n+5}{3n^5-7}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$

(c) $\sum_{n=1}^{\infty} \frac{1}{(-2)^n(n^2+1)}$

(d) $\sum_{n=1}^{\infty} \frac{3^n}{n!}$

7. Determine if the series is absolutely convergent, conditionally convergent, or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{2n+1} n^4}{3^{n-1}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^5}{(-10)^{n+1}}$

(c) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

(d) $\sum_{n=1}^{\infty} \frac{(-2)^n n!}{5 \cdot 8 \cdot 11 \cdots (3n+2)}$