Math 152 Exam 3 Review

The following is a collection of questions to review the topics for the second exam. This is not intended to represent an actual exam nor does it have every type of problem seen int he homework.
These questions cover sections 11.4, 11.5, 11.6, 11.8, 11.9, 11.10, 11.11

Important Maclaurin series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + ... \qquad |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + ... \qquad R = \infty$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - ... \qquad R = \infty$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... \qquad R = \infty$$

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - ... \qquad R = 1$$

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1} = x - \frac{x^2}{2} + \frac{x^3}{3} - ... \qquad -1 < x \leq 1$$

1. Determine if the series converges.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-5)^{n+1} n^4}{9^{n+3}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{7n^3 + \cos(2n)}{n^4 + 1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{3^{2n} - 7}$

2. Suppose that the power series $\sum_{n=0}^{\infty} c_n(x-2)^n$ has a radius of convergence of 7. What can be concluded about the convergence/divergence of the following pair of series?

$$\text{(I)} \sum_{n=0}^{\infty} (-1)^n c_n 4^n \qquad \text{(II)} \sum_{n=0}^{\infty} c_n 9^n$$

3. Consider the 5th partial sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n7^n}$ as an approximation. Use the alternating series rule to obtain an upper bound on the absolute value of the error.

4. Assume that the series $\sum_{n=1}^{\infty} (-1)^{n+1} b_n$ will converge by the alternating series test.

Which of these is an approximation to the sum of the series so that the maximum error will be less than 0.001 and contains the fewest number of terms?

(a) $b_1 + b_2 + b_3 + b_4 + b_5$

(b) $b_1 - b_2 + b_3 - b_4 + b_5$

(c) $-b_1 + b_2 - b_3 + b_4 - b_5$

(d) $b_1 + b_2 + b_3 + b_4 + b_5 + b_6$

(e) $b_1 - b_2 + b_3 - b_4 + b_5 - b_6$

(f) $-b_1 + b_2 - b_3 + b_4 - b_5 + b_6$

n	b_n
1	0.5
2	0.0625
3	0.01388889
4	0.00390625
5	0.00125
6	0.00043403
7	0.00015944
8	0.00006104

Is the approximation more or less than the actual sum?

(a) more (b) less

5. Determine if the series is absolutely convergent, conditionally convergent, or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{n^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/5}}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{7\sin(n^2+1)}{n^4-2n+5}$

6. Find the interval and radius of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n}}{n^2\ 25^n}$.

7. Find the radius of convergence and the interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x+4)^n}{n 7^n}$$

8. Find the interval and radius of convergence of the series $\sum_{n=1}^{\infty} \frac{n!(2x-3)^n}{5^n}$.

9. Find the sum of these series

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (5)^{2n+1}}{(2n+1)!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+1}}{3^{2n}(2n)!}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-2)^n}{n!}$

10. Find the Maclaurin series for the function $f(x) = \dfrac{x^2}{(1-3x)^2}$.

11. Use a MacLaurin series for $f(x) = x^3 \arctan(5x^2)$ to answer the following.

(a) $f'(x) =$

(b) $\int f(x)dx =$

12. Find the 23th derivative of $f(x)$ at $x = 5$, i.e. $f^{(23)}(5)$, for $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^n(n+2)}(x-5)^n$

13. Find the Taylor series for $f(x) = \dfrac{1}{x^2}$ about $a = 5$. Express your answer in summation notation.

14. Find the Taylor series for $f(x) = (x+1)e^x$ about $a = 2$. Express your answer in summation notation.

15. Find the 4th degree Taylor polynomial, $T_4(x)$, for $f(x) = x\ln(x)$ about $x = 6$.