Week in Review–Additional Chapter 5 Material

Section 5.2: Matrix Multiplication

- if A is a mxn matrix and B is a nxp matrix then the matrix AB has a size of mxp
  - note: the number of columns of A equals the number of rows of B.
- identity matrix I_n
  - size nxn
  - all zeros except for $a_{1,1} = a_{2,2} = a_{3,3} = ... = 1$

$$A = \begin{bmatrix} 7 & 2 & 4 \\ 6 & 5 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 9 & 3 & 0 \\ -1 & 2 & 8 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 3 \\ -2 & 5 \\ 2 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} \text{x} & 1 \\ 2 & 5 \end{bmatrix}$$

1. Use the above matrices to compute the following.

   (a) $AC =$

   (b) $BD =$

   (c) $DB =$

   (d) $D^2 =$

   (e) $2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 & 8 \\ 2 & 3 & 0 \\ 0 & 1 & 5 \end{bmatrix} =$

   (f) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 & 8 \\ 2 & 3 & 0 \\ 0 & 1 & 5 \\ 8 & 2 & 5 \end{bmatrix} =$

2. A dietitian plans a meal around two foods. The number of units of vitamin A and vitamin C in each ounce of these foods is represented by the matrix M.

$$M = \begin{array}{c} \\ \text{Vitamin A} \\ \text{Vitamin C} \end{array} \begin{array}{cc} \text{Food I} & \text{Food II} \\ \left[\begin{array}{c} 30 \\ 7 \end{array}\right. & \left.\begin{array}{c} 90 \\ 45 \end{array}\right] \end{array} \qquad B = \begin{array}{cc} \text{Food I} & \text{Food II} \\ [\ 5 & 2\ ] \end{array} \qquad L = \begin{array}{cc} \text{Food I} & \text{Food II} \\ [\ 9 & 4\ ] \end{array}$$

   The matrices B and L represent the amount of each food (in ounces) consumed by the girl at breakfast and lunch, respectively. Explain the meaning of the entries in these computations..

   (a) $LM = \begin{bmatrix} 298 & 990 \end{bmatrix}$

   (b) $MB^T = \begin{bmatrix} 330 \\ 125 \end{bmatrix}$

Section 5.3: The inverse of a Matrix

- the matrix must be square.
- NOT all square matrices have an inverse.
- the inverse of A is denoted A^{-1}
- $AA^{-1} = A^{-1}A = I$
- A system of equations may be written as a matrix equation: AX=B
  - A is the coefficient matrix
  - X is the variable matrix
  - If A has an inverse, then the solution is $X = A^{-1}B$.
    - Matrix A not having an inverse does not imply that the system of equations has no solution. It means that you need to try another method to solve the problem.

3. If $A = \begin{bmatrix} 5 & 1 \\ 5 & 2 \end{bmatrix}$ find A^{-1}

4. If $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ find A^{-1}

5. If $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 2 & 3 \\ 2 & 1 & 2 \end{bmatrix}$ find A^{-1}

6. Determin if the matrices A and B are inverses of each other.
   $A = \begin{bmatrix} 1 & 2 & \text{x} \\ 0 & 5 & 2 \\ 0 & 2 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 2x-2 & 4-5x \\ 0 & 1 & -2 \\ 0 & -2 & 5 \end{bmatrix}$

7. Answer the following using this system of equations.
   $$\begin{aligned} 2x \quad\; + z &= 2 \\ 2x + y - z &= 1 \\ 3x + y - z &= 4 \end{aligned}$$

   (a) Write down the coefficient matrix.

   (b) Write the system of equations as a matrix equation.

   (c) Solve the system of equations using matrices.

8. True or False. A system of equations is represented by the matrix equation $AX = B$. If the coefficient matrix, A, does not have an inverse then the system of equations does not have a solution.

9. Solve for the matrix X. Assume that all matrices are square and all needed inverses are possible.

   (a) $BX = E - CX$

   (b) $XJ + XA = K$