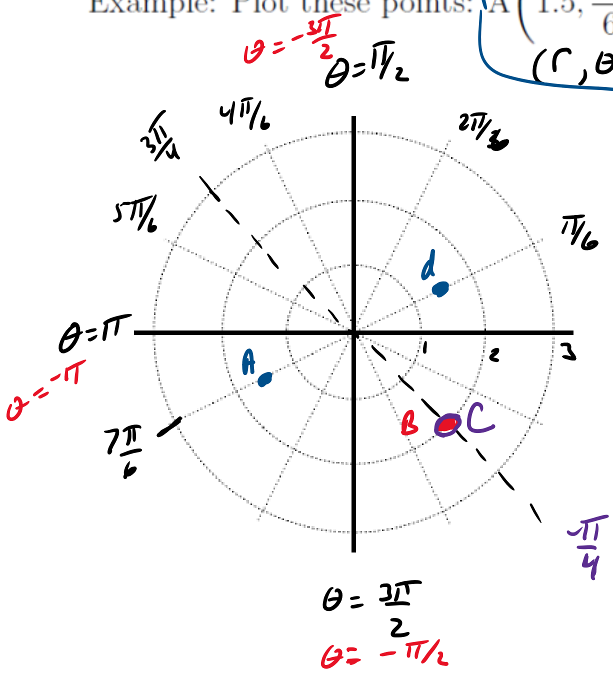
Section 10.3: Polar Coordinates

Definition: The **polar coordinate system** is a way to reference the points in the xy-plane where each point is of the form (r, θ). r is the distance from the point in the plane to the **pole** (or origin). θ is the angle from the **polar axis** (positive x-axis) to the line segment connecting the point and the origin. This angle is positive when measured in the counterclockwise direction and negative when measured in the clockwise direction.

Example: Plot these points: $A\left(1.5, \frac{7\pi}{6}\right)$, $B\left(-2, \frac{3\pi}{4}\right)$, $C\left(2, \frac{-\pi}{4}\right)$.

(r, θ) (r, θ)

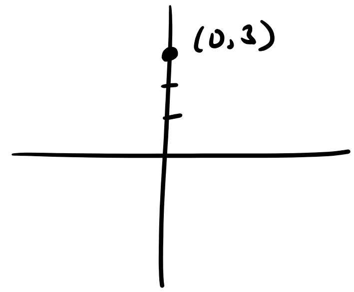
$d\left(-1.5, \frac{7\pi}{6}\right) = \left(1.5, \frac{\pi}{6}\right)$



$\theta = 0, 2\pi, 4\pi, 6\pi, 8\pi$

Example: Give a polar coordinate of the Cartesian point $(\overset{x}{0}, \overset{y}{3})$.

$$\left(3, \frac{\pi}{2}\right) \equiv \left(-3, \frac{3\pi}{2}\right) \equiv \left(-3, -\frac{\pi}{2}\right)$$

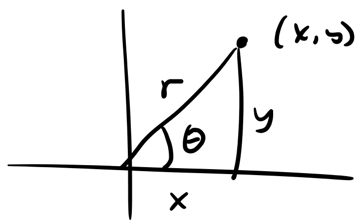


Converting Between Polar and Cartesian Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2+y^2}$



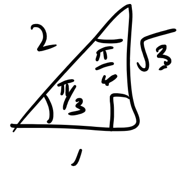
$\tan\theta = \frac{y}{x}$

$\theta = \arctan \frac{y}{x}$

Pg 3

Wednesday, November 20, 2019 4:04 PM

Example: Convert the point $\left(1.5, \frac{\pi}{6}\right)$ from polar to Cartesian coordinates.



$x = r\cos\theta$ $\qquad y = r\sin\theta$

$x = 1.5\cos\left(\frac{\pi}{6}\right)$ $\qquad y = 1.5\sin\left(\frac{\pi}{6}\right)$

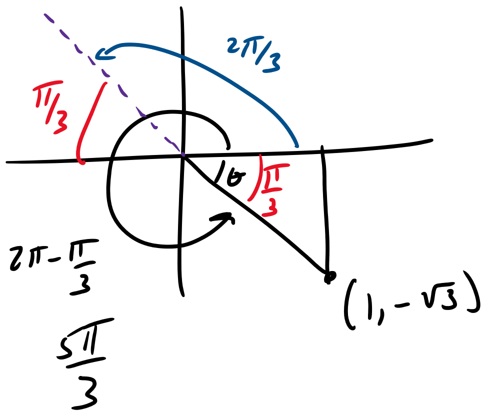
$x = 1.5\left(\frac{\sqrt{3}}{2}\right)$ $\qquad y = 1.5\left(\frac{1}{2}\right) = \frac{3}{2}\cdot\frac{1}{2} = \frac{3}{4}$

$= \frac{3}{2}\cdot\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$

point $\left(\frac{3\sqrt{3}}{4}, \frac{3}{4}\right)$

Example: Convert the point $(1, -\sqrt{3})$ from Cartesian to polar coordinates.

x y

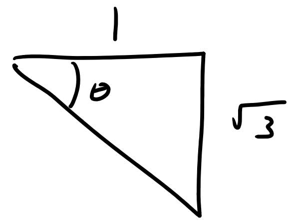


$r = \sqrt{1^2 + (-\sqrt{3})^2}$

$r = \sqrt{1+3}$

$r = \sqrt{4}$

$r = 2$



$\tan\theta = \frac{\sqrt{3}}{1}$

$\theta = \arctan(\sqrt{3})$

$\theta = \frac{\pi}{3}$

$$\left(2, \frac{5\pi}{3}\right) \equiv \left(2, \frac{-\pi}{3}\right) \equiv \left(-2, \frac{2\pi}{3}\right)$$

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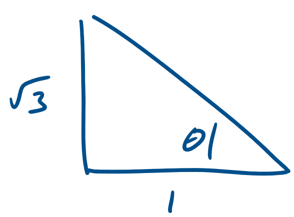
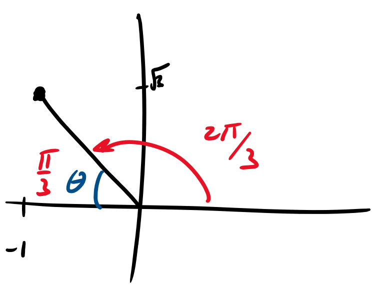
point

$(-1, \sqrt{3})$ $\qquad r = 2$



$\tan\theta = \frac{\sqrt{3}}{1}$

$\theta = \pi/3$

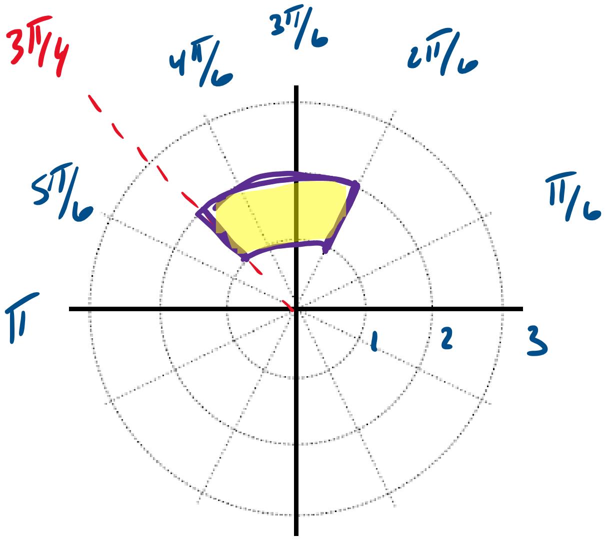


$\left(2, \frac{2\pi}{3}\right)$

Pg 4

Wednesday, November 20, 2019 4:04 PM

Example: Sketch the region in the plane consisting of points whose polar coordinates satisfies these conditions: $1 \le r \le 2$, $\pi/3 \le \theta \le 3\pi/4$



Pg 5

Wednesday, November 20, 2019 4:04 PM

Example: Find a Cartesian equations for the polar equation and sketch the graph.

$r = 10\cos\theta$

$x = r\cos\theta$

$y = r\sin\theta$

$r^2 = x^2 + y^2$

$r = 10\cos\theta$

$r^2 = 10\underbrace{r\cos\theta} \longrightarrow x^2 + y^2 = 10x$

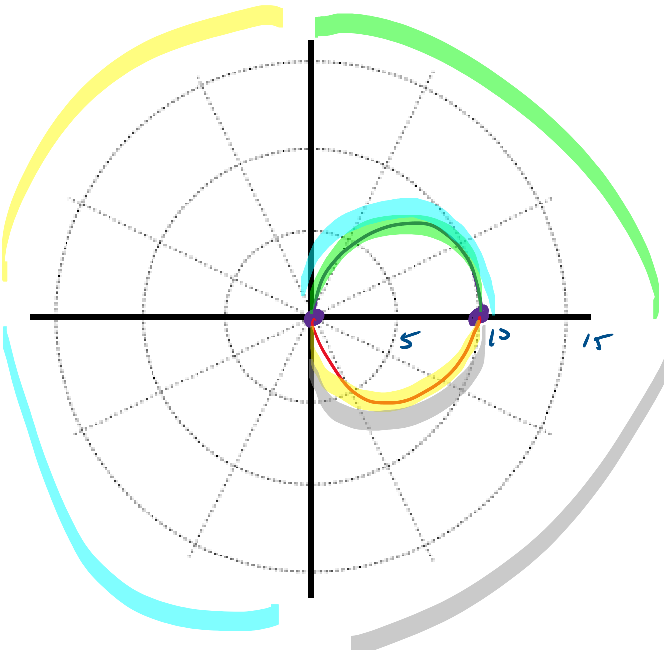
$x^2 - 10x + y^2 = 0$

$x^2 - 10x + (5)^2 + y^2 = 0 + (5)^2$

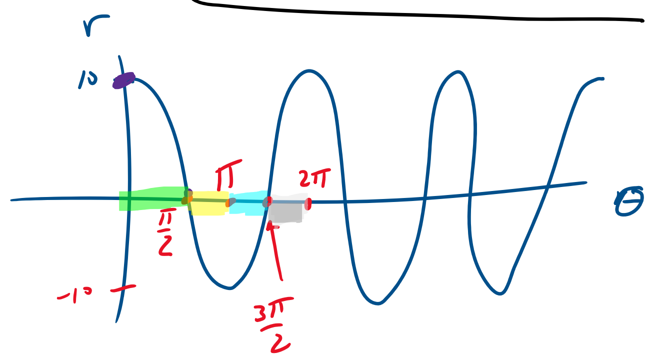
$(x-5)^2 + y^2 = 25$

Center $(5, 0)$

Radius is 5



$r = 10\cos\theta$

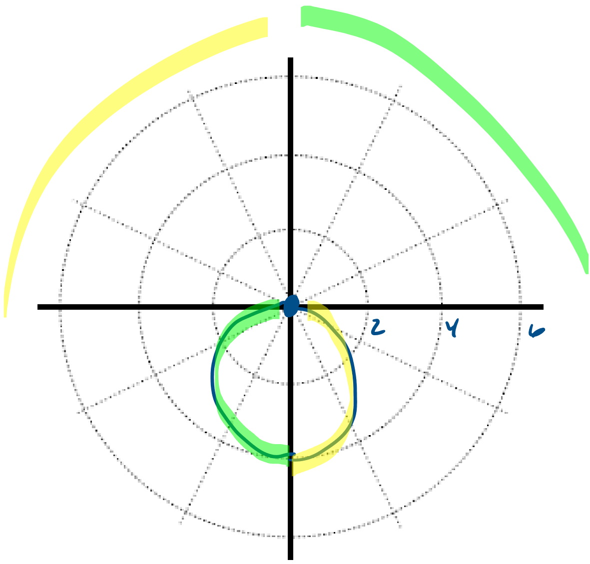


$x = r\cos\theta$
$y = r\sin\theta$

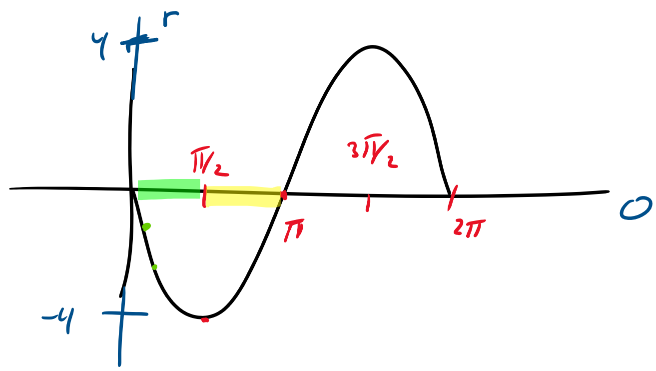
Example: Find a Cartesian equations for the polar equation and sketch the graph.

$r = -4\sin\theta \longrightarrow r^2 = -4r\sin\theta \longrightarrow x^2 + y^2 = -4y$

Circle
Radius $= 2 = \frac{4}{2}$



$r = -4\sin\theta$



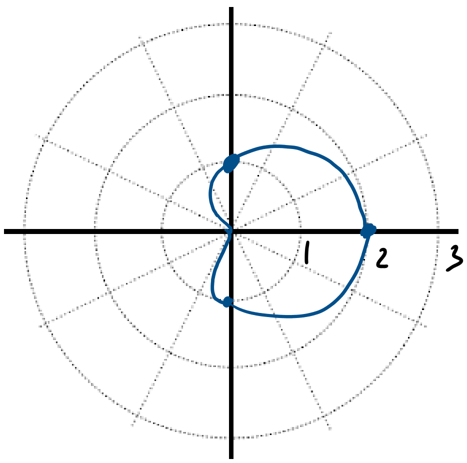
Wednesday, November 20, 2019 4:04 PM

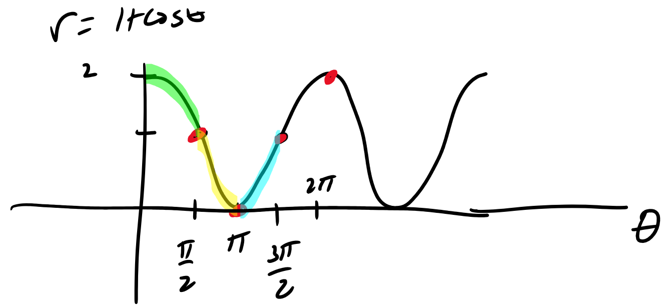
$x = r\cos\theta$
$y = r\sin\theta$

$r^2 = x^2 + y^2$

Example: Find a Cartesian equations for the polar equation and sketch the graph.

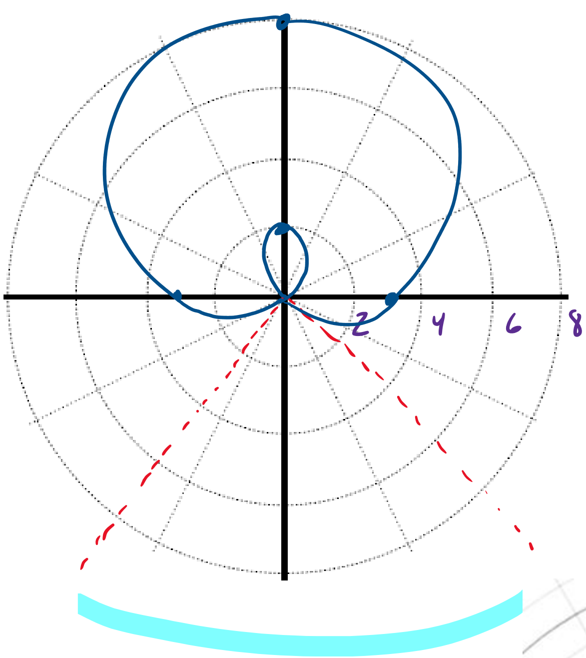
$r = 1 + \cos\theta \longrightarrow r^2 = r + r\cos\theta \longrightarrow \boxed{x^2 + y^2 = \sqrt{x^2+y^2} + x}$



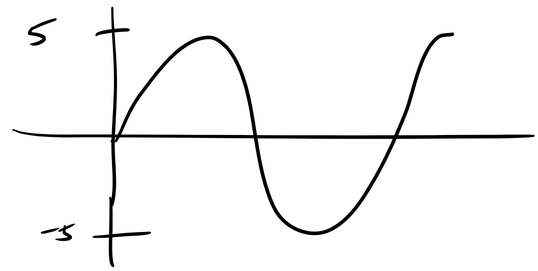


Example: Sketch the graph of the limacon.

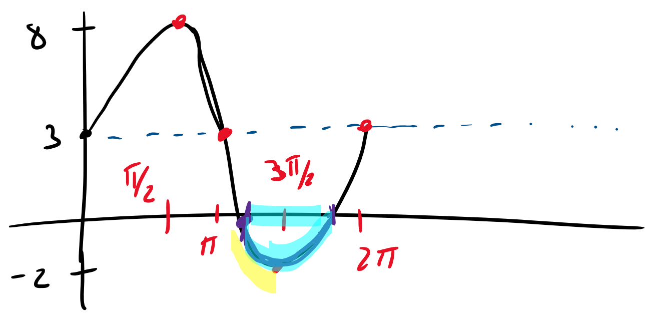
$r = 3 + 5\sin\theta$

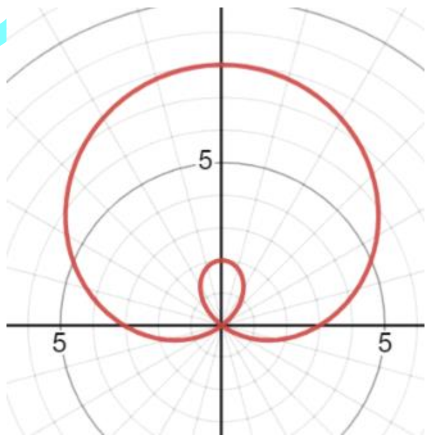


$r = 5\sin\theta$



$r = 3 + 5\sin\theta$





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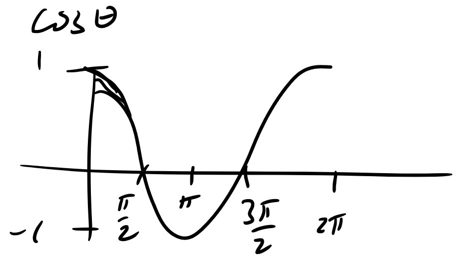
Wednesday, November 20, 2019 4:10 PM

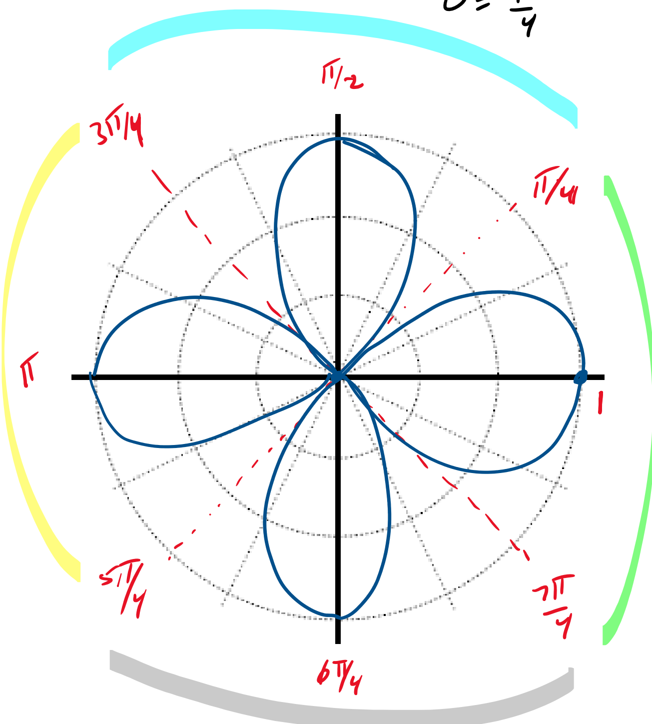
Example: Sketch the graph of the rose.

$r = \cos 2\theta$

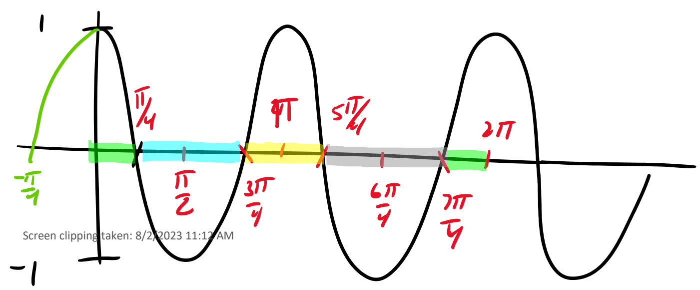
$2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$





$r = \cos(2\theta)$



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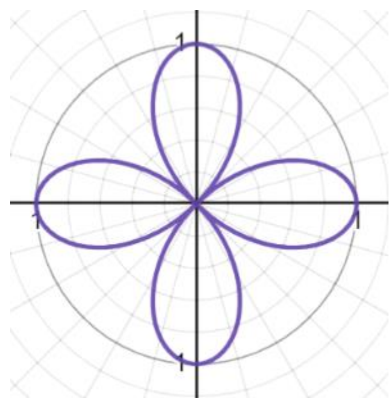
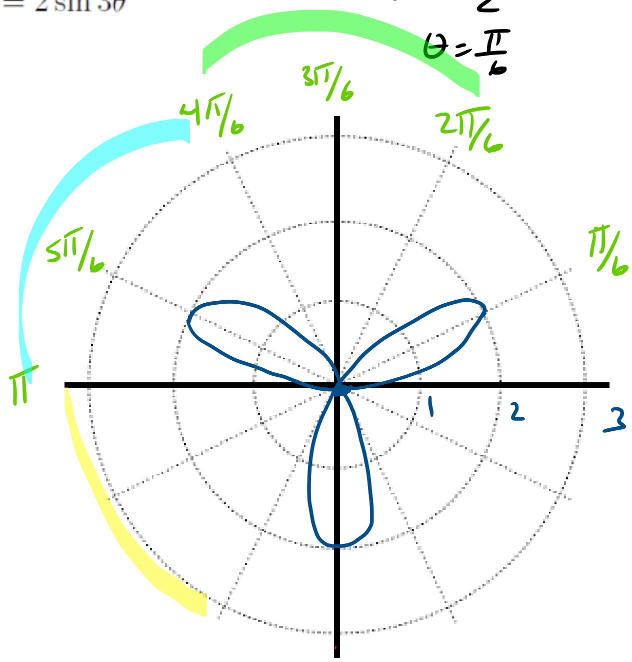


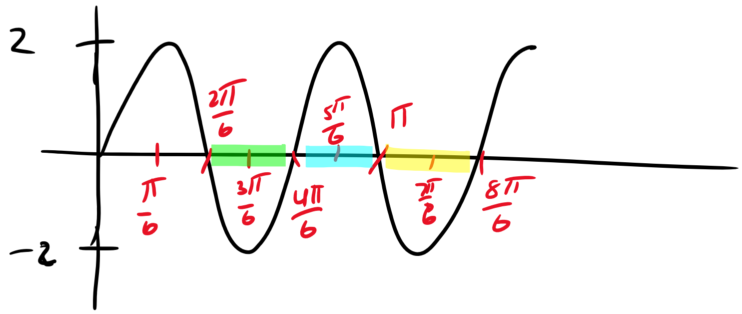
Example: Sketch the graph of the rose.

$r = 2\sin 3\theta$

$3\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{6}$





Note: Here is a link that gives some of the conditions for the number of loops in the polar graph.

https://en.wikipedia.org/wiki/Rose_(mathematics)

Example: Find a polar equation for the Cartesian equations.

$x = r\cos\theta$
$y = r\sin\theta$
$x^2 + y^2 = r^2$

$y^2 = 5x$

$(r\sin\theta)^2 = 5r\cos\theta$

$r^2\sin^2\theta = 5r\cos\theta$

$r\sin^2\theta = 5\cos\theta \longrightarrow r = \frac{5\cos\theta}{\sin^2\theta}$