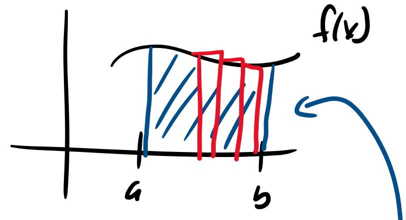
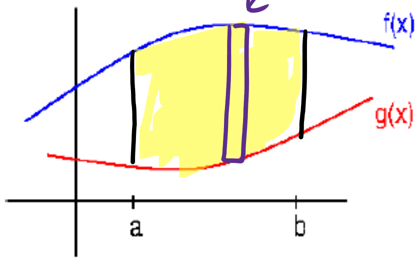
if $f(x) \geq 0$




$$\int_a^b f(x)\,dx = \text{Area}$$

Consider the continuous functions $f(x)$ and $g(x)$ with the property on the interval $[a, b]$ that both are above the x-axis and $\underline{f(x) \geq g(x)}$. Write down the computation that will give the area bounded between these functions on this interval.



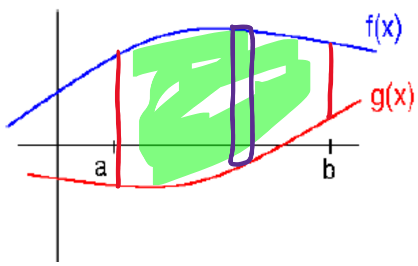

height = $f(x) - g(x)$

$$\sum_{i=1}^{n} [f(x_i) - g(x_i)]\,\Delta x$$

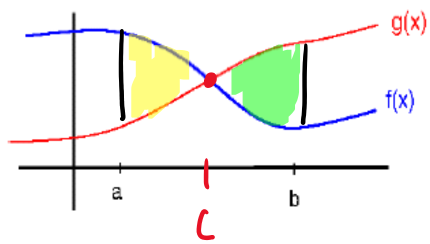
Limit process

$$\text{Area} = \int_a^b f(x) - g(x)\,dx = \int_a^b (\text{top} - \text{bottom})\,dx$$

For the next graphs, set-up the integral(s) that will give the area that is bounded between $f(x)$ and $g(x)$ on the interval $[a, b]$ .




$$\int_a^b f(x) - g(x)\,dx$$

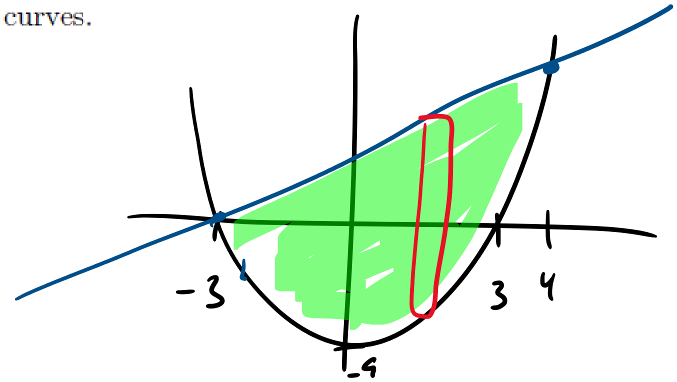



$$\text{Area} = \int_a^c f(x) - g(x)\,dx + \int_c^b g(x) - f(x)\,dx$$

$$\int_a^b |f(x) - g(x)|\,dx$$ ← no credit for this style of answer.

Example: Find the area that is bounded by these curves.

$y = x + 3$
$y = x^2 - 9$




$x^2 - 9 = x + 3$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

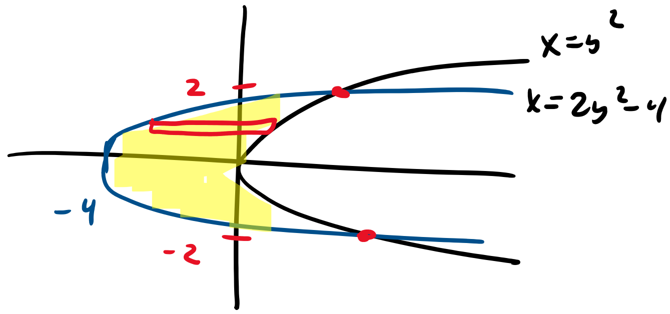
$x = 4 \quad x = -3$

$$\text{Area} = \int_{-3}^{4} x + 3 - (x^2 - 9)\, dx = \int_{-3}^{4} x + 3 - x^2 + 9 \, dx$$

$$= \int_{-3}^{4} x - x^2 + 12 \, dx$$

Example: Find the area that is bounded by these curves.

$x = y^2$
$x = 2y^2 - 4$




$y^2 = 2y^2 - 4$

$4 = y^2$

$y = \pm 2$

"height" = Right − Left.

$$\int_{-2}^{2} y^2 - (2y^2 - 4)\, dy = \int_{-2}^{2} y^2 - 2y^2 + 4\, dy = \int_{-2}^{2} 4 - y^2\, dy$$

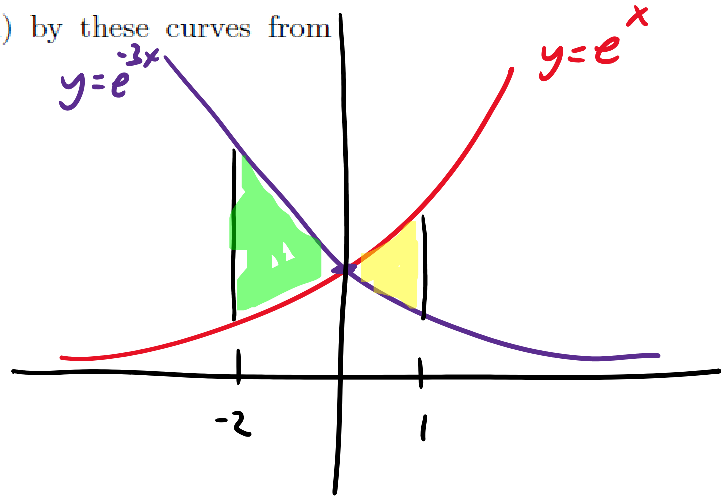
$$= 4y - \frac{y^3}{3} \Big|_{-2}^{2} = 8 - \frac{8}{3} - \left(-8 + \frac{8}{3}\right)$$

$$= 8 - \frac{8}{3} + 8 - \frac{8}{3} = 16 - \frac{16}{3} = \frac{32}{3}$$

$$2\int_{0}^{2} y^2 - (2y^2 - 4)\, dy$$

Example: Find the area that is bounded(enclosed) by these curves from $x = -2$ to $x = 1$.

$y = e^{-3x}$
$y = e^{x}$




$$\int_{-2}^{0} e^{-3x} - e^{x}\,dx + \int_{0}^{1} e^{x} - e^{-3x}\,dx$$

$$= \left(\frac{1}{-3}e^{-3x} - e^{x}\right)\Big|_{-2}^{0} + \left(e^{x} - \frac{1}{-3}e^{-3x}\right)\Big|_{0}^{1}$$

$$= -\frac{1}{3} - 1 - \left(-\frac{1}{3}e^{6} - e^{-2}\right) + e^{1} + \frac{1}{3}e^{-3} - \left(1 + \frac{1}{3}\right)$$

Example: Set up the integral(s), with respect to the variable $y$, that gives the area that is bounded(enclosed) by these curves.

$y = 2\sqrt{x}$ $\longrightarrow$ $x = \frac{y^2}{4}$

$y = \frac{-x}{3}$ $\longrightarrow$ $x = -3y$

$3x + y = 16$ $\longrightarrow$ $x = \frac{16-y}{3}$

Intersection $y = \frac{-x}{3}$, $3x+y=16$

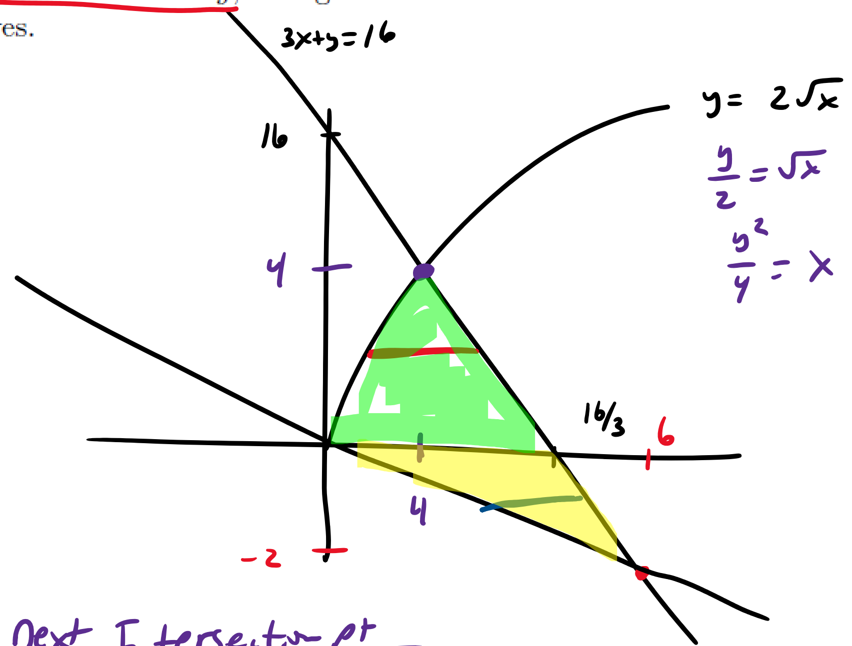
$3x + \frac{-x}{3} = 16$

$9x - x = 48$

$8x = 48$

$x = 6$

$y = -2$




Next Intersection pt

$3\left(\frac{y^2}{4}\right) + y = 16$

$\frac{3y^2}{4} + y = 16$

$3y^2 + 4y = 64$

$3y^2 + 4y - 64 = 0$

$(3y+16)(y-4) = 0$

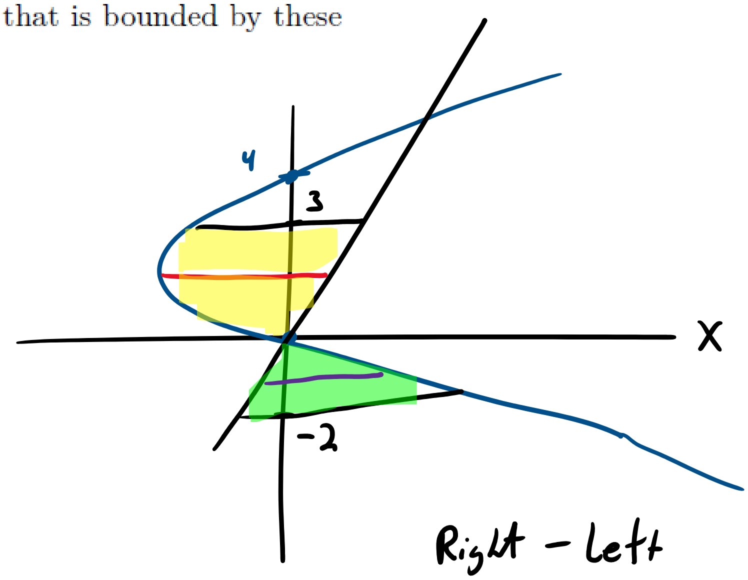
$y = \frac{-16}{3}$ $\quad$ $y = 4$

$$\boxed{\int_{-2}^{0} \frac{16-y}{3} - (-3y)\, dy + \int_{0}^{4} \frac{16-y}{3} - \frac{y^2}{4}\, dy}$$

Example: Set up the integral(s) that will give area that is bounded by these curves on the interval $-2 \leq y \leq 3$.

$\rightarrow$ $x = y^2 - 4y = y(y-4)$

$y = 0.5x$ $\rightarrow$ $y = \frac{1}{2}x \rightarrow x = 2y$

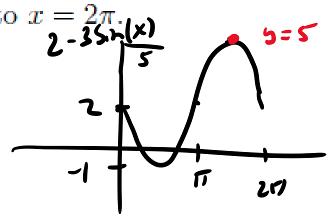


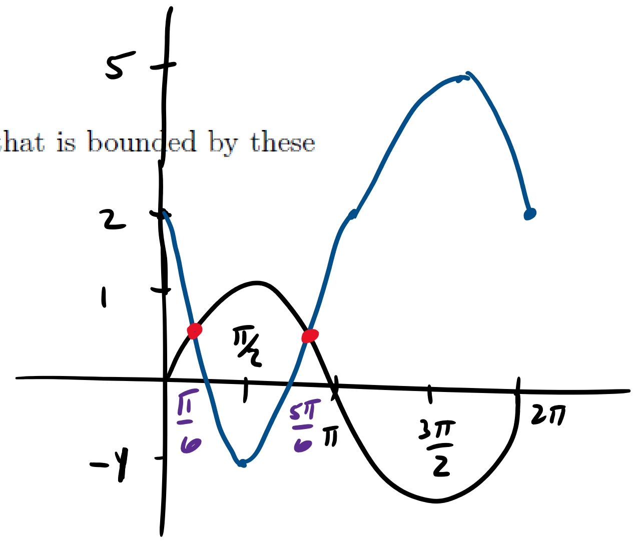

Right − Left

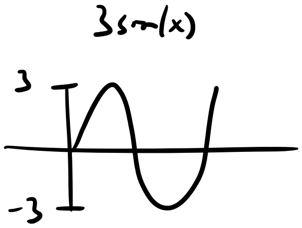
$$\int_{-2}^{0} y^2 - 4y - 2y \, dy + \int_{0}^{3} 2y - (y^2 - 4y) \, dy$$

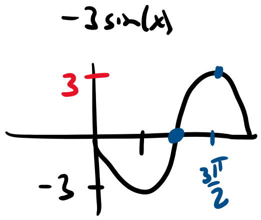
Example: Set up the integral(s) that will give area that is bounded by these curves from $x = 0$ to $x = 2\pi$.

$y = \sin(x)$
$y = 2 - 3\sin(x)$



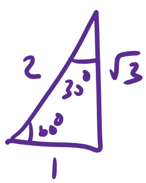




$$\sin(x) = 2 - 3\sin(x)$$

$$4\sin(x) = 2$$

$$\sin(x) = \frac{1}{2}$$

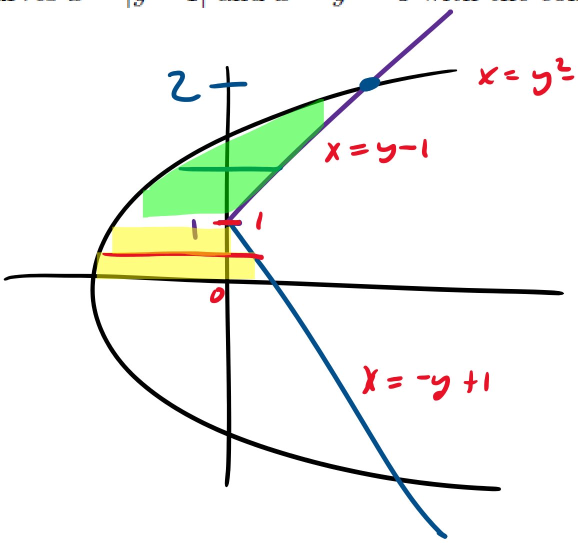



$$x = \frac{\pi}{6} \qquad x = \frac{5\pi}{6}$$

$$\boxed{\int_0^{\pi/6} 2 - 3\sin(x) - \sin(x)\, dx + \int_{\pi/6}^{5\pi/6} \sin(x) - (2 - 3\sin(x))\, dx + \int_{5\pi/6}^{2\pi} 2 - 3\sin(x) - \sin(x)\, dx}$$

$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Example: Set up the integral(s) that will give area that is bounded by these curves $x = |y - 1|$ and $x = y^2 - 3$ with the condition that $y \geq 0$




$x = |y - 1| = \begin{cases} y - 1 & y \geq 1 \\ -(y - 1) & y < 1 \end{cases}$

$y \geq 1$

$x = y - 1$

$y = x + 1$

$y < 1$

$x = -(y - 1)$

$x = -y + 1$

$y = 1 - x$

Intersection

$y = x + 1 \quad x = y^2 - 3$

$y - 1 = x$

$y - 1 = y^2 - 3$

$0 = y^2 - y - 2$

$0 = (y - 2)(y + 1)$

$y = 2 \quad y = -1$

$$\int_0^1 -y + 1 - (y^2 - 3)\,dy + \int_1^2 y - 1 - (y^2 - 3)\,dy$$