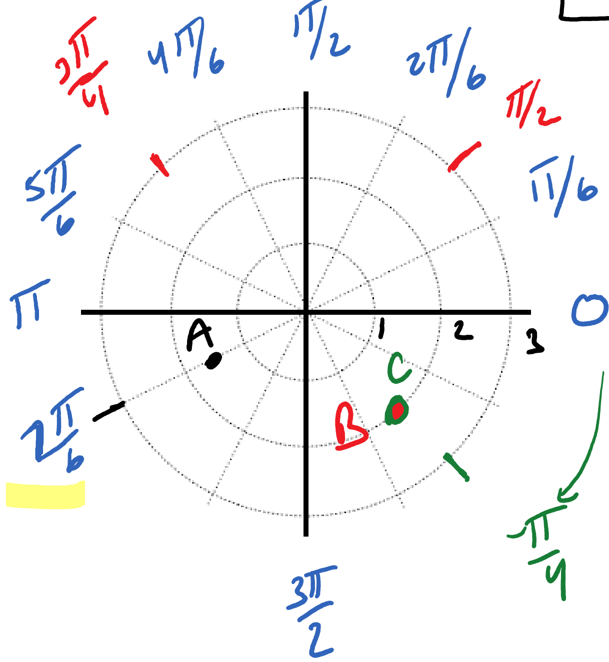
Section 10.3: Polar Coordinates

Definition: The **polar coordinate system** is a way to reference the points in the xy-plane where each point is of the form (r, θ). r is the distance from the point in the plane to the **pole** (or origin). θ is the angle from the **polar axis** (positive x-axis) to the line segment connecting the point and the origin. This angle is positive when measured in the counterclockwise direction and negative when measured in the clockwise direction.

$r \quad \theta$

Example: Plot these points: $A\left(1.5, \frac{7\pi}{6}\right)$, $B\left(-2, \frac{3\pi}{4}\right)$, $C\left(2, \frac{-\pi}{4}\right)$.

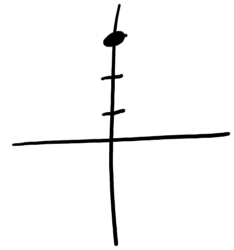


$x \quad y$

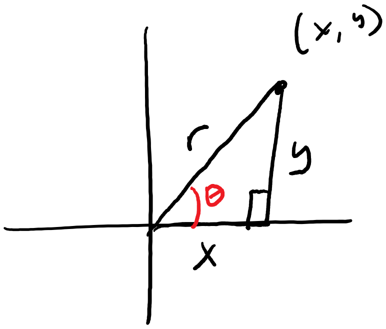
Example: Give a polar coordinate of the Cartesian point $(0, 3)$.

$\left(3, \frac{\pi}{2}\right) \qquad \left(-3, \frac{3\pi}{2}\right)$

$\left(3, \frac{\pi}{2} + 2\pi\right) \qquad \left(-3, \frac{-\pi}{2}\right)$



Converting Between Polar and Cartesian Coordinates:



$r^2 = x^2 + y^2$

$r = \sqrt{x+y}$

$\tan\theta = \frac{y}{x}$

$\theta = \arctan\left(\frac{y}{x}\right)$

$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$\cos\theta = \frac{x}{r} \qquad \sin\theta = \frac{y}{r}$

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

$r \quad \theta$

Example: Convert the point $\left(1.5, \frac{\pi}{6}\right)$ from polar to Cartesian coordinates.

$x = r\cos\theta$

$x = 1.5\cos\left(\frac{\pi}{6}\right)$

$x = 1.5\left(\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{4}$

$y = r\sin\theta$

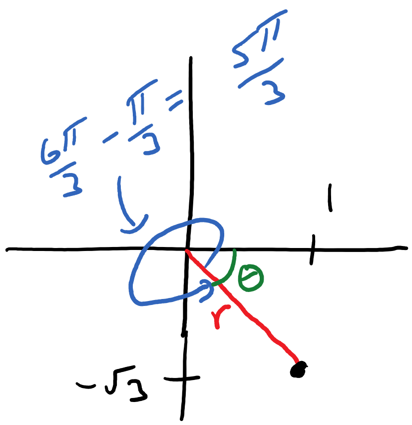
$y = 1.5\sin\left(\frac{\pi}{6}\right)$

$y = \frac{3}{2}\left(\frac{1}{2}\right) = \frac{3}{4}$

$\boxed{\left(\frac{3\sqrt{3}}{4}, \frac{3}{4}\right)}$

$x \quad y$

Example: Convert the point $(1, -\sqrt{3})$ from Cartesian to polar coordinates.



$r = \sqrt{(1)^2 + (-\sqrt{3})^2}$

$= \sqrt{1+3}$

$= \sqrt{4}$

$= 2$

$\tan\theta = \left|\frac{-\sqrt{3}}{1}\right|$

$\tan\theta = \sqrt{3}$

reference angle

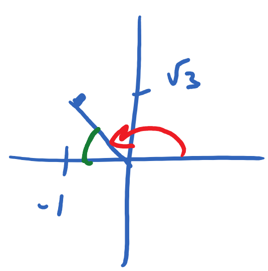
$\theta = \arctan(\sqrt{3})$

$\theta = \pi/3$

$\left(2, \frac{-\pi}{3}\right)$

$\left(2, \frac{5\pi}{3}\right)$

$(-1, \sqrt{3})$



$r = 2$

$\left(2, \frac{2\pi}{3}\right)$

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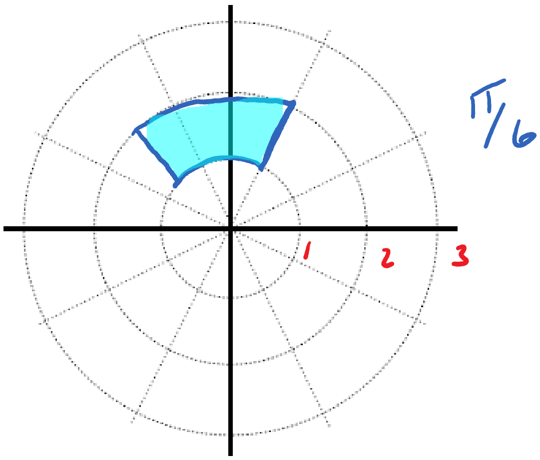
Example: Sketch the region in the plane consisting of points whose polar coordinates satisfies these conditions: $1 \leq r \leq 2$, $\pi/3 \leq \theta \leq 3\pi/4$

$2\pi/6 = \frac{\pi}{3}$



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$r^2 = x^2 + y^2 \qquad x = r\cos\theta \qquad y = r\sin\theta$

Example: Find a Cartesian equations for the polar equation and sketch the graph.

$\boxed{r = 10\cos\theta} \longrightarrow r^2 = 10r\cos\theta \longrightarrow x^2 + y^2 = 10x$

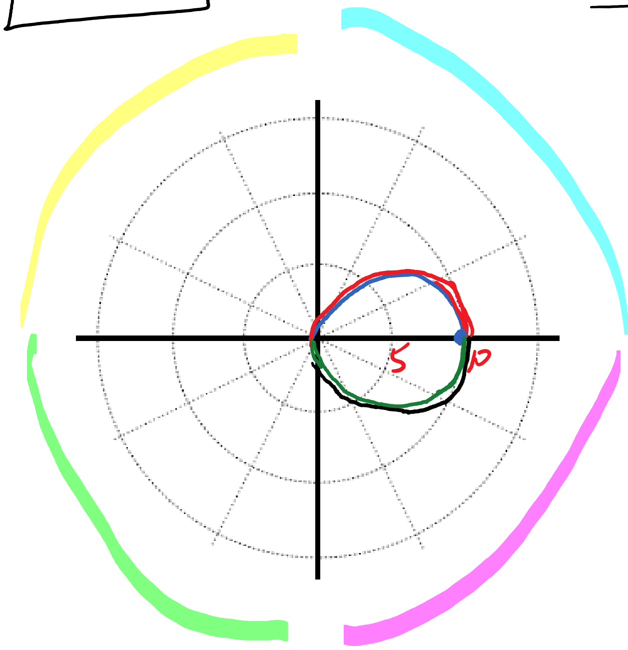
$x^2 - 10x + y^2 = 0$

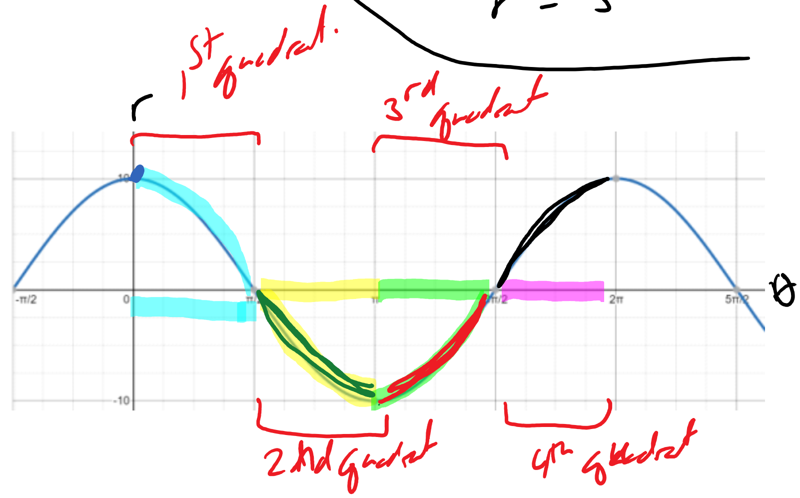
$x^2 - 10x + 5^2 + y^2 = 5^2$

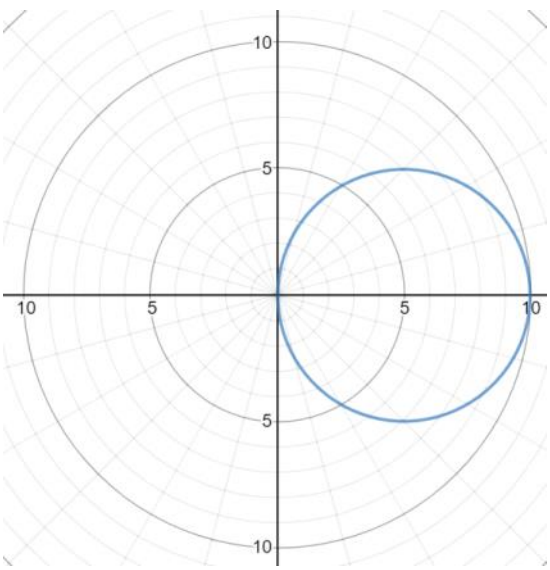
$(x-5)^2 + y^2 = 25$

center $(5,0)$

$r = 5$





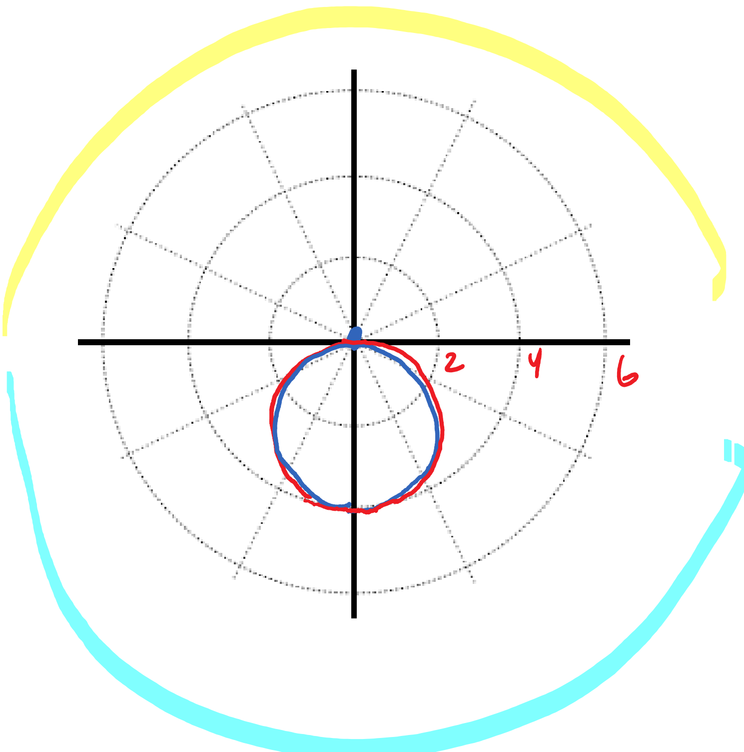


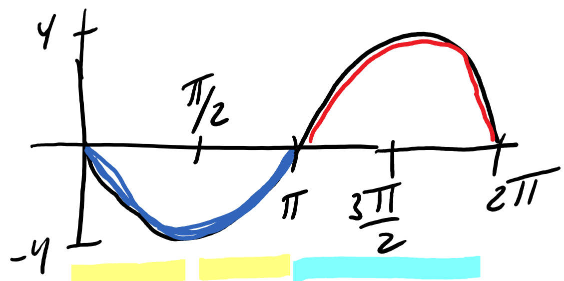
Example: Find a Cartesian equations for the polar equation and sketch the graph.

$r = -4\sin\theta$

$r^2 = -4r\sin\theta$

$$\boxed{x^2 + y^2 = -4y}$$





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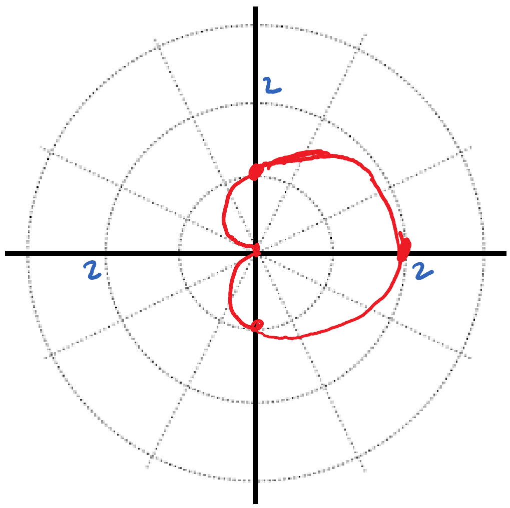
Example: Find a Cartesian equations for the polar equation and sketch the graph.

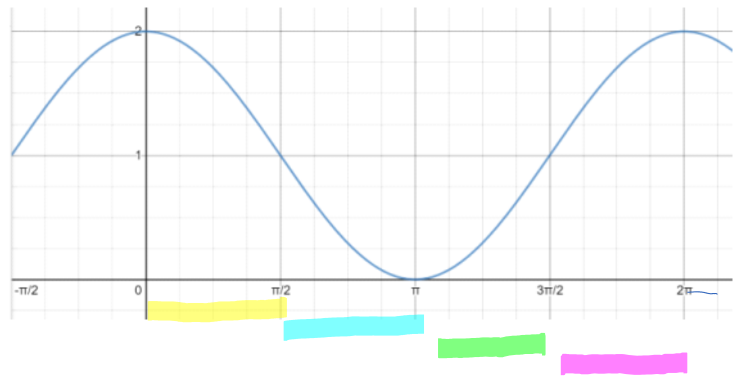
$r = 1 + \cos\theta$

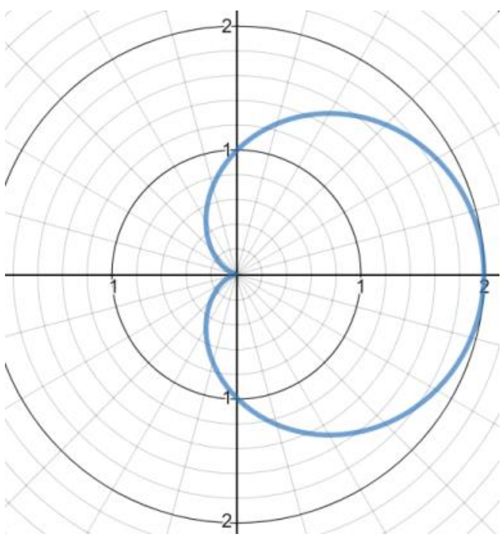
$r^2 = r + r\cos\theta$

$$\boxed{x^2 + y^2 = \sqrt{x^2+y^2} + x}$$



$r = 1 + \cos\theta$





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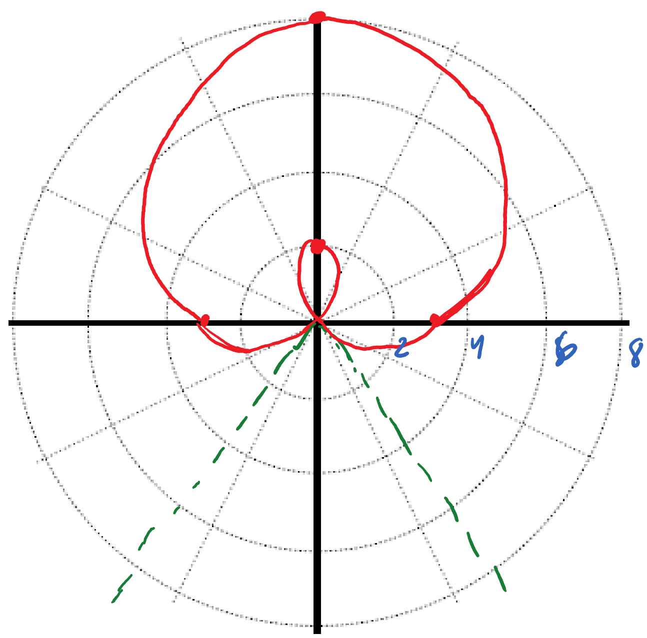
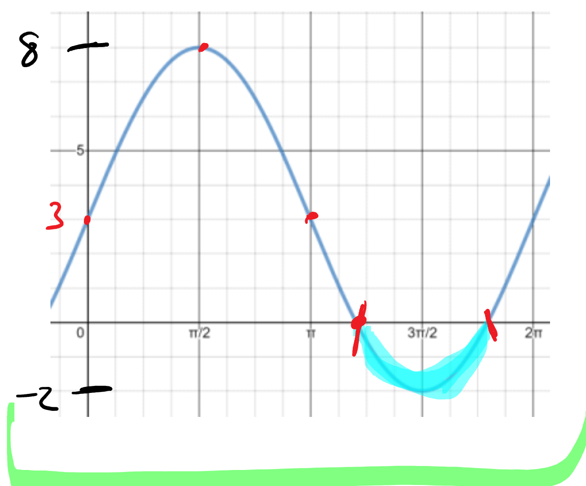
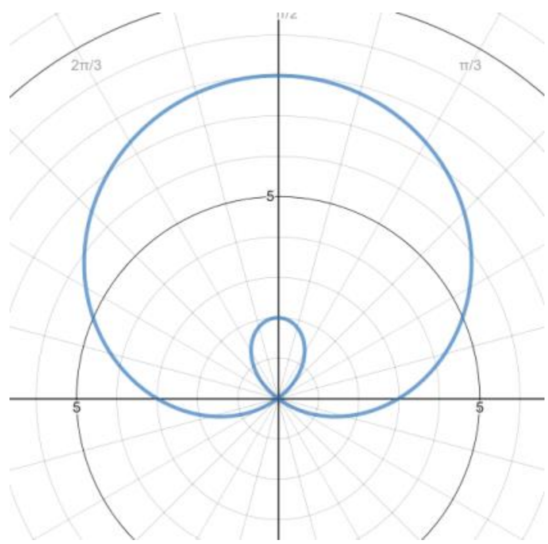
Example: Sketch the graph of the limacon.

$r = 3 + 5\sin\theta$



$r = 3 + 5\sin\theta$





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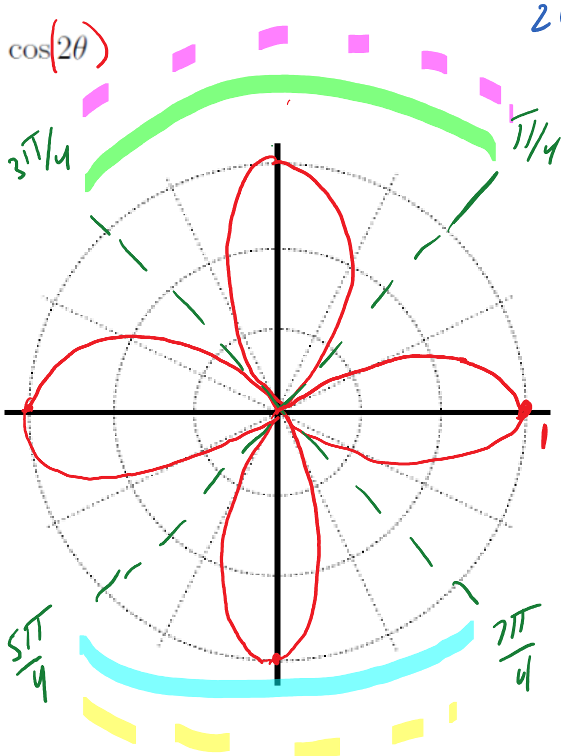
Example: Sketch the graph of the rose.

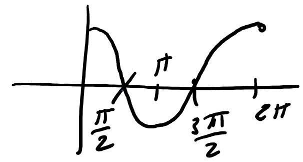
$r = \cos(2\theta)$

$2\theta = \frac{\pi}{2}$

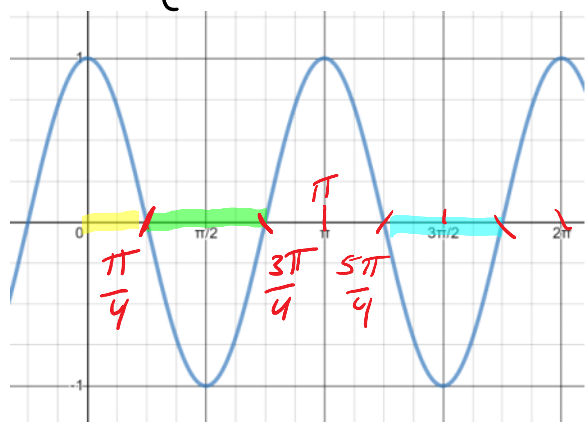
$\theta = \frac{\pi}{4}$

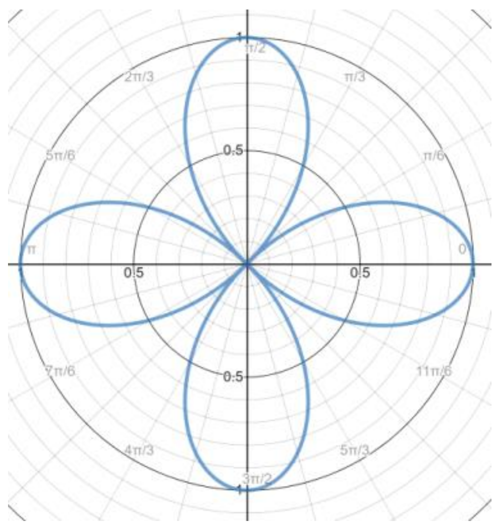


$r = \cos\theta$



$r = \cos(2\theta)$

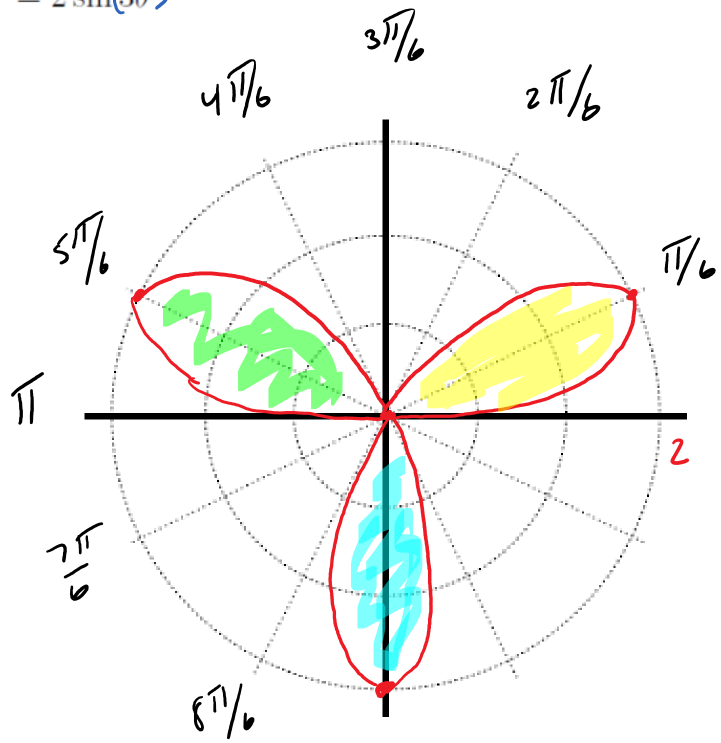


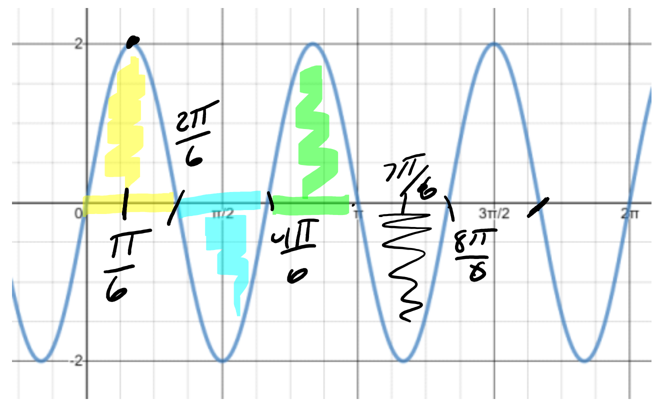


$3\theta = \pi/2 \qquad \theta = \pi/6$

Example: Sketch the graph of the rose.

$r = 2\sin(3\theta)$





Note: Here is a link that gives some of the conditions for the number of loops in the polar graph.

https://en.wikipedia.org/wiki/Rose_(mathematics)

Example: Find a polar equation for the Cartesian equations.

$x = r\cos\theta$

$y = r\sin\theta$

$y^2 = 5x$

$(r\sin\theta)^2 = 5r\cos\theta$

$r^2\sin^2\theta = 5r\cos\theta$

$r\sin^2\theta = 5\cos\theta$

$r = \dfrac{5\cos\theta}{\sin^2\theta}$