Math 152 Week in review sections 7.1, 7.2 and Exam 1 Review

Solutions and questions can be found at the link:
https://www.math.tamu.edu/~kahlig/152WIR.html

$$\int u\,dV = uV - \int V\,du$$

Week in Review: Sections 7.1 and 7.2

Compute the following integrals

1. $\int x\sin(x)\,dx$ $= -x\cos(x) - \int -\cos(x)\,dx$

$= -x\cos(x) + \sin(x) + C$

	D		I
$u =$	x	$+$	$\sin(x) = dV$
$du =$	1	$-\int$	$-\cos(x) = V$

D		I
x	$+$	$\sin(x)$
1	$-$	$-\cos(x)$
0	$+\int$	$-\sin(x)$

$\int x\sin(x)\,dx = -x\cos(x) + \sin(x) + \int 0\,dx$

$= -x\cos(x) + \sin(x) + C$

2. $\int x^3 e^{2x}\, dx$

$$= \frac{x^3}{2} e^{2x} - \frac{3x^2}{4} e^{2x} + \frac{6x}{8} e^{2x} - \frac{6}{16} e^{2x} + C$$

D		I
x^3	$+$	e^{2x}
$3x^2$	$-$	$\frac{1}{2} e^{2x}$
$6x$	$+$	$\frac{1}{4} e^{2x}$
6	$-$	$\frac{1}{8} e^{2x}$
0	$+$	$\frac{1}{16} e^{2x}$
	$\int$	

3. $\int 3x^2 \ln(x)\, dx$

D		I
$\ln(x)$	$+$	$3x^2$
$\frac{1}{x}$	$-\int$	x^3

$$= x^3 \ln(x) - \int \frac{1}{x} x^3 \, dx$$

$$= x^3 \ln(x) - \int x^2 \, dx$$

$$= x^3 \ln(x) - \frac{x^3}{3} + C$$

4. $\int \cos(x)e^{5x}\,dx = J$

	D		I
	e^{5x}	$+$	$\cos(x)$
	$5e^{5x}$	$-$	$\sin(x)$
	$25e^{5x}$	$+\int$	$-\cos(x)$

$$J = \sin x\, e^{5x} + 5\cos(x)e^{5x} - 25\int \cos(x)\,e^{5x}\,dx$$

$$J = \underbrace{\sin(x)\,e^{5x} + 5\cos(x)e^{5x}} - 25J$$

$$26J = [\quad]$$

$$J = \frac{1}{26}\left[\sin(x)\,e^{5x} + 5\cos(x)e^{5x}\right] + C$$

Integral	Method
$\int \cos^2(x)\ dx$	$\cos^2(x) = \frac{1}{2}(1+\cos(2x))$
$\int \cos^n(x)\sin(x)\ dx$	$u = \cos(x)$
$\int \tan^n(x)\sec^2(x)\ dx$	$u = \tan(x)$

Integral	Method
$\int \sin^2(x)\ dx$	$\sin^2(x) = \frac{1}{2}(1-\cos(2x))$
$\int \sin^n(x)\cos(x)\ dx$	$u = \sin(x)$
$\int \sec^n(x)\tan(x)\sec(x)\ dx$	$u = \sec(x)$

sometimes useful trig identities.

$$\sin(A)\cos(B) = \frac{1}{2}[\sin(A-B)+\sin(A+B)]$$

$$\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B)-\cos(A+B)]$$

$$\cos(A)\cos(B) = \frac{1}{2}[\cos(A-B)+\cos(A+B)]$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

Integrate the following.

1. $\int \cos^3(2x)\,dx = \int \cos^2(2x)\cos(2x)\,dx$

$= \int [1 - \sin^2(2x)] \cos(2x)\,dx = \int (1-u^2)\frac{1}{2}\,du$

$u = \sin(2x)$

$du = 2\cos(2x)\,dx$

$\frac{1}{2}du = \cos(2x)\,dx$

$= \frac{1}{2}\int 1-u^2\,du$

$= \frac{1}{2}\left[u - \frac{u^3}{3}\right] + C$

$= \frac{1}{2}\left[\sin(2x) - \frac{1}{3}\sin^3(2x)\right] + C$

$\sin(2x)^3 \neq (\sin(2x))^3 = \sin^3(2x)$

2. $\int \tan^3(x)\,dx = \int \tan(x)\tan^2(x)\,dx$

$$= \int \tan(x)\left[\sec^2(x)-1\right]dx$$

$$= \int \underline{\tan(x)\sec^2(x)} - \tan(x)\,dx$$

$$= \int \tan(x)\sec^2(x)\,dx - \int \tan(x)\,dx$$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$$= \int u\,du - \int \frac{\sin(x)}{\cos(x)}\,dx$$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$-du = \sin(x)\,dx$

$$= \frac{1}{2}u^2 - -\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\tan^2(x) + \ln|u| + C$$

$$= \frac{1}{2}\tan^2(x) + \ln|\cos(x)| + C$$

3. $\int \frac{\sin^3(x)}{\sec^2(x)}\,dx = \int \sin^3(x)\cos^2(x)\,dx$

$$= \int \sin^2(x)\cos^2(x)\,\underline{\sin(x)\,dx}$$

$$= \int (1-\cos^2(x))\cos^2(x)\sin(x)\,dx$$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$-du = \sin(x)\,dx$

$$= \int -[1-u^2]\,u^2\,du$$

$$= \int -u^2 + u^4\,du$$

$$= -\frac{u^3}{3} + \frac{u^5}{5} + C$$

$$= -\frac{1}{3}\cos^3(x) + \frac{1}{5}\cos^5(x) + C$$

4. $\int \cos^2(x)\sin^2(x)\,dx = \int \frac{1}{2}\left[1+\cos(2x)\right]\cdot\frac{1}{2}\left[1-\cos(2x)\right]\,dx$

$= \frac{1}{4}\int (1+\cos(2x))(1-\cos(2x))\,dx$

$= \frac{1}{4}\int 1 - \cos^2(2x)\,dx$

$\cos^2(\theta) = \frac{1}{2}\left[1+\cos(2\theta)\right]$

$= \frac{1}{4}\int 1 - \frac{1}{2}\left[1+\cos(4x)\right]\,dx$

$= \frac{1}{4}\int 1 - \frac{1}{2} - \frac{1}{2}\cos(4x)\,dx$

$= \frac{1}{4}\int \frac{1}{2} - \frac{1}{2}\cos(4x)\,dx$

$= \frac{1}{4}\left[\frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{4}\sin(4x)\right] + C$

5. $\int \sec^4(x)\tan^3(x)\,dx$ $= \int \sec^2(x)\tan^3(x)\,\underline{\sec^2(x)\,dx}$

$= \int (\tan^2 x + 1)\tan^3(x)\sec^2(x)\,dx$

$\boxed{\begin{array}{l} u = \tan(x) \\ du = \sec^2(x)\,dx \end{array}}$

$= \int (u^2+1)\,u^3\,du = \int u^5 + u^3\,du$

$= \frac{1}{6}u^6 + \frac{1}{4}u^4 + C$

$= \frac{1}{6}\tan^6(x) + \frac{1}{4}\tan^4(x) + C$

$\int \sec^4(x)\tan^3(x)\,dx = \int \sec^4(x)\tan^2(x)\tan(x)\,dx$

$= \int \sec^3(x)\left[\sec^2(x) - 1\right]\sec(x)\tan(x)\,dx$

$u = \sec(x)$

$du = \sec(x)\tan(x)\,dx$

$= \int u^3\left[u^2 - 1\right]du$

$= \int u^5 - u^3\,du = \frac{1}{6}u^6 - \frac{1}{4}u^4 + C$

$= \frac{1}{6}\sec^6(x) - \frac{1}{4}\sec^4(x) + C$

The following is a collection of questions to review the topics for the first exam. This is not intended to represent an actual exam nor does it have every type of problem seen in the homework.
These questions cover sections 5.5, 6.1, 6.2, 6.3 and 6.4.

1. Compute $\int x^5 \sqrt[3]{x^3 - 10}\, dx$ $= \int x^3 (x^3 - 10)^{1/3}\, x^2\, dx$

$u = x^3 - 10 \rightarrow u + 10 = x^3$

$du = 3x^2\, dx$

$\frac{1}{3} du = x^2\, dx$

$= \int (u + 10)\, u^{1/3} \cdot \frac{1}{3}\, du$

$= \frac{1}{3} \int u^{4/3} + 10\, u^{1/3}\, du$

$= \frac{1}{3} \left[\frac{3}{7} u^{7/3} \quad \frac{3}{4} \;\; 10 \cdot 3 u^{4/3} \right]$

$= \frac{1}{3} \left[\frac{3}{7} (x^3 - 10)^{7/3} + \frac{30}{4} (x^3 - 10)^{4/3} \right] + C$

2. Rewrite the integral $\displaystyle\int_{-1}^{4} \frac{x}{(x+7)^3}\,dx$ using an appropriate substitution.

$u = x + 7$

$du = dx$

$u - 7 = x$

$x = 4 \rightarrow u = 11$

$x = -1 \rightarrow u = 6$

$$\boxed{\int_{6}^{11} \frac{u-7}{u^3}\,du}$$

3. A particle is moved along the x−axis by a force, in newtons, of $f(x) = \dfrac{1+x}{x^2+1}$ at a point that is x meters from the origin. Find the work done moving the particle from $x = 0$ to $x = 1$.

$$W = \int_0^1 \frac{1+x}{x^2+1}\,dx = \int_0^1 \frac{1}{x^2+1}\,dx + \int_0^1 \frac{x}{x^2+1}\,dx$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

$$= \arctan(x)\Big|_0^1 + \int_{x=0}^{x=1} \frac{1}{2}\,\frac{1}{u}\,du$$

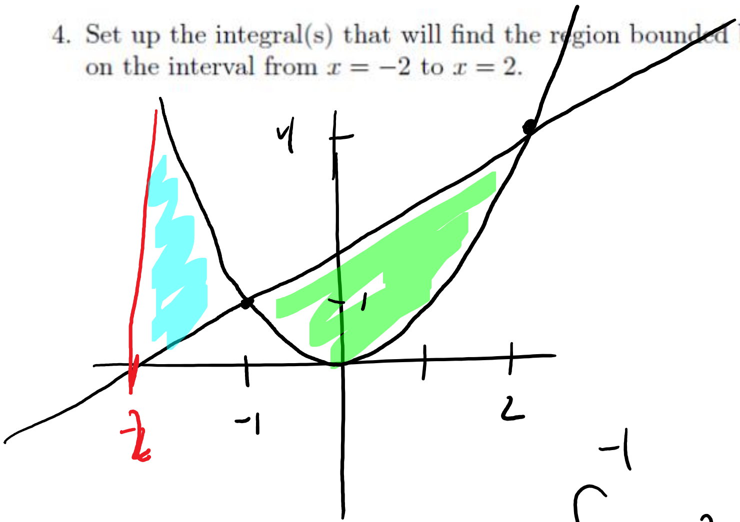
$$= \arctan(1) - \arctan(0) + \frac{1}{2}\ln|u|\Big|_{x=0}^{x=1}$$

$$= \arctan(1) - \arctan(0) + \frac{1}{2}\ln(x^2+1)\Big|_0^1$$

$$= \arctan(1) - \underbrace{\arctan(0)}_{0} + \frac{1}{2}\ln(2) - \underbrace{\frac{1}{2}\ln(1)}_{0}$$

$$= \frac{\pi}{4} + \frac{1}{2}\ln(2)$$

4. Set up the integral(s) that will find the region bounded by $y = x^2$, $y = x + 2$, on the interval from $x = -2$ to $x = 2$.



$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad x = -1$

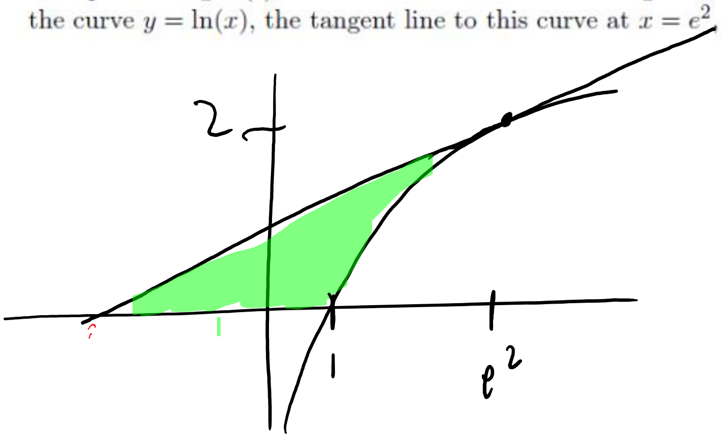
$$\int_{-2}^{-1} x^2 - (x+2)\,dx + \int_{-1}^{2} x+2 - x^2\,dx$$

dy Integral

5. Set up the integral(s) that will find the area of the region that is enclosed by the curve $y = \ln(x)$, the tangent line to this curve at $x = e^2$, and the $x-$axis.

point $(e^2, \ln(e^2))$

$(e^2, 2)$



$y' = \frac{1}{x}$ $\quad m_{tan} = \frac{1}{e^2}$

$y - 2 = \frac{1}{e^2}(x - e^2$

$y - 2 = \frac{1}{e^2}x - 1$

$y - 1 = \frac{1}{e^2}x$

$e^2(y-1) = x$

$y = \ln(x)$

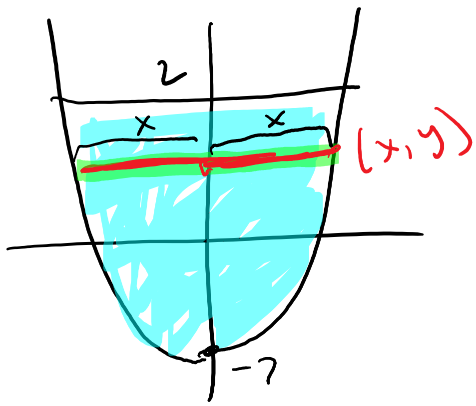
$x = e^y$

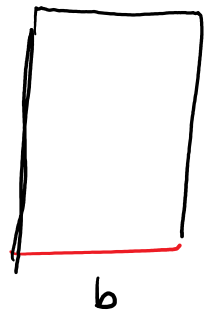
$$A = \int_0^2 e^y - e^2(y-1)\,dy$$

$$= \int_0^2 e^y - e^2 y + e^2\,dy$$

6. The base of a solid is the region enclosed by $y = x^2 - 7$ and $y = 2$. Cross-sections perpendicular to the $y-$axis are rectangles where the height is twice the length of the base. Setup the integral to find the volume of the solid.

$y + 7 = x^2$





$h = 2b = 4x$

$b = 2x$

$A = (2x)(4x)$

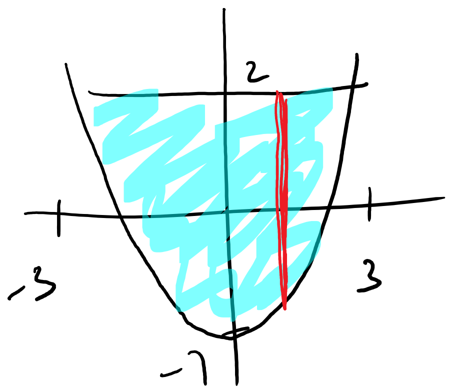
$A = 8x^2$

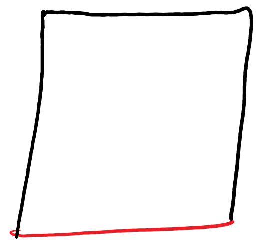
$A = 8(y+7)$

dy Integral

$$V = \int_{-7}^{2} 8(y+7)\, dy$$

7. The base of a solid is the region enclosed by $y = x^2 - 7$ and $y = 2$. Cross-sections perpendicular to the x−axis are rectangles where the height is twice the length of the base. Setup the integral to find the volume of the solid.





$h = 2b = 2(9 - x^2)$

$b = 2 - (x^2 - 7)$

$b = 2 - x^2 + 7$

$b = 9 - x^2$

$A = b\,h$

$= (9 - x^2)\,2(9 - x^2)$

$= 2(9 - x^2)^2$

dx Integral

$x^2 - 7 = 2$

$x^2 = 9$

$x = \pm 3$

$$V = \int_{-3}^{3} 2(9 - x^2)^2\,dx$$

$$= 2\int_{0}^{3} 2(9 - x^2)^2\,dx$$

8. Which of the following represents the area between the curves $y = \dfrac{12}{x}$ and $y = 6$ on the interval from $x = 1$ to $x = 4$?

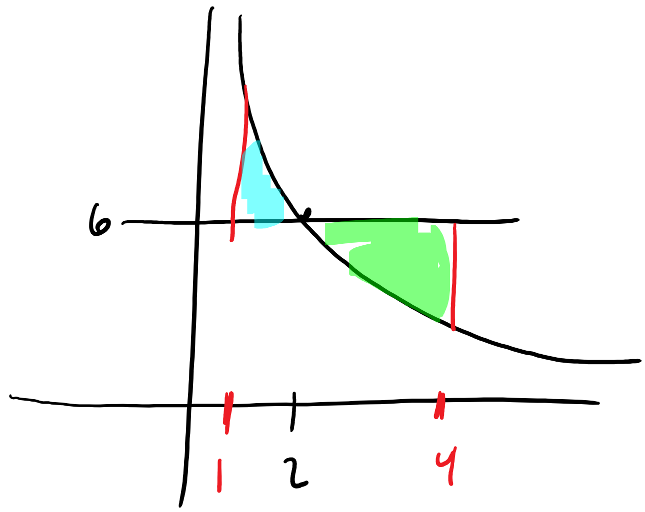
(a) $\int_1^2 \left(6 - \frac{12}{x}\right) dx + \int_2^4 \left(\frac{12}{x} - 6\right) dx$

(b) $\int_3^6 \left(4 - \frac{12}{y}\right) dy + \int_6^{12} \left(\frac{12}{y} - 1\right) dy$

(c) $\int_1^4 \left(\frac{12}{x} - 6\right) dx$

(d) $\int_3^{12} \left(\frac{12}{y} - 1\right) dy$

(e) $\int_1^4 \left(6 - \frac{12}{x}\right) dx$



9. The region bounded by the curves $x = 1$, $y = 2$ and $xy = 8$ is rotated about the line $y = -1$.

$xy = 8$

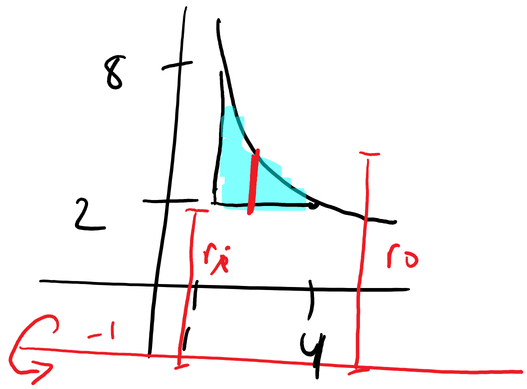
$y = \frac{8}{x}$

(a) Using **disk/washers** set up an integral(s) that represent this volume.

dx Integral

$r_i = 2 - (-1) = 3$

$r_o = \frac{8}{x} - (-1) = \frac{8}{x} + 1$



$$V = \int_1^4 \pi \left[\left(\frac{8}{x} + 1\right)^2 - (3)^2 \right] dx$$

(b) Using **cylindrical shells** set up an integral(s) that represent this volume.

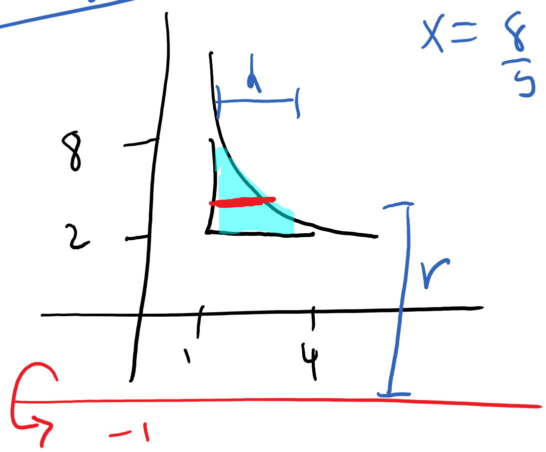
dy Integral

$xy = 8$

$x = \frac{8}{y}$

$r = y - (-1) = y + 1$

$h = \frac{8}{y} - 1$



$$V = \int_2^8 2\pi (y+1)\left(\frac{8}{y} - 1\right) dy$$

10. Set up the integral(s) that would find the volume of the solid obtained by rotating the region bounded the curves

$y = 3x - x^2 \qquad y = 3x - 9$

$3x - x^2 = 3x - 9$

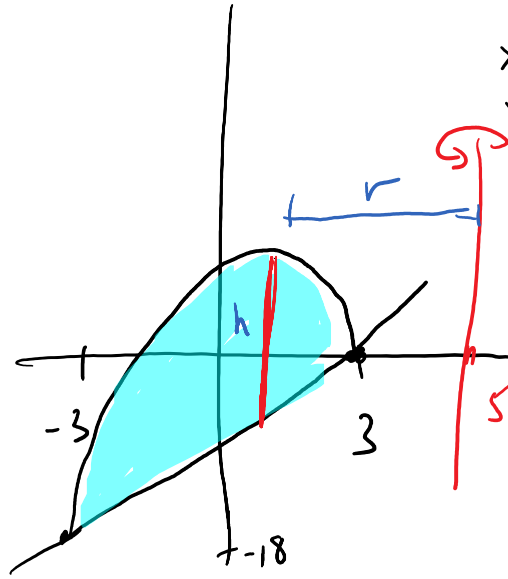
$-x^2 = -9$

$x^2 = 9$

$x = \pm 3$

(a) about the line $x = 5$.

shell



$r = 5 - x$

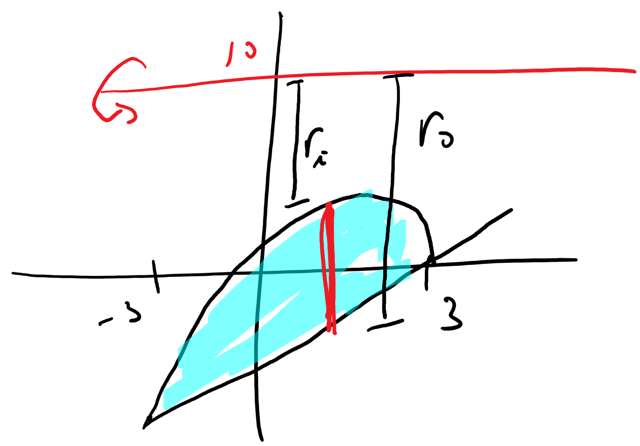
$h = 3x - \cancel{x}^2 - (3x - 9)$

$= 9 - x^2$

$$V = \int_{-3}^{3} 2\pi (5-x)(9-x^2)\,dx$$

(b) about the line $y = 10$.

washer



$r_o = 10 - (3x - 9)$

$= 10 - 3x + 9$

$= 19 - 3x$

$r_i = 10 - (3x - x^2)$

$r_i = 10 - 3x + x^2$

$$V = \int_{-3}^{3} \pi \left[(19 - 3x)^2 - (10 - 3x + x^2)^2 \right] dx$$

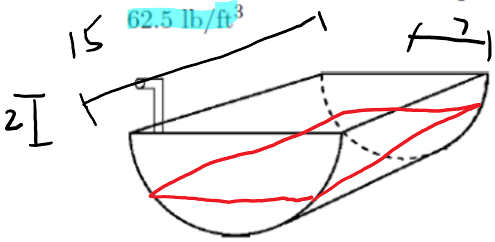
$$V = \int_{-3}^{1} \pi \left[(19 - 3x) - (10 - 3x + x^2) \right] dx$$

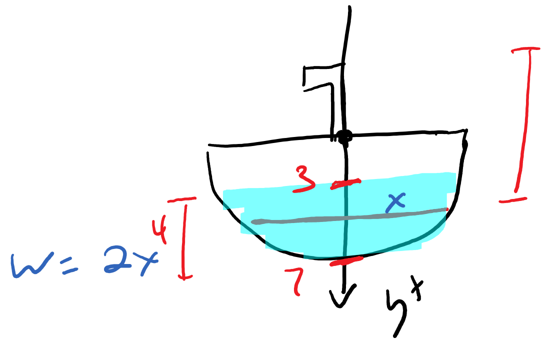
11. A trough is in the shape of a half cylinder(on its side). The length of the trough is 15 ft and it has a radius of 7 ft. Assuming that the trough if filled with water to a depth of 4 ft. There is a spout at the top of the tank that is 2 ft tall.

Set up (but do not evaluate) an integral that will compute the work required to pump all the water out of the spout.

Be sure to indicate on the picture where you are placing the axis and the direction of the positive axis. Use the fact that water weighs 62.5 lb/ft^3

$\rho g = 62.5$





$d = y + 2$

$w = 2x$

$x^2 + y^2 = 7^2$

$x^2 = 49 - y^2$

$x = \sqrt{49 - y^2}$

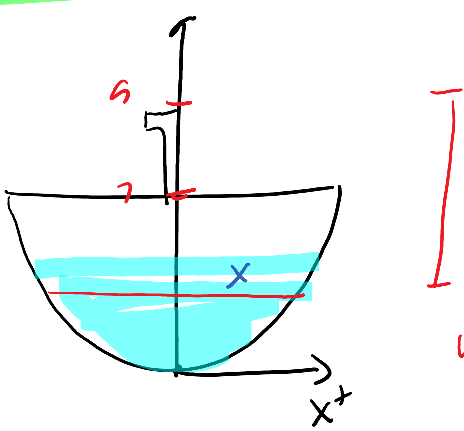
$V = lw\,\Delta y$

$= 15(2x)\,\Delta y$

$V = 30x\,\Delta y$

$V = 30\sqrt{49 - y^2}\,\Delta y$

$$\boxed{W = \int_3^7 \rho g\,(y+2)\,30\sqrt{49 - y^2}\,dy}$$



$d = 9 - y$

$w = 2x$

$(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-7)^2 = 7^2$

x^+

$(x-0)^2 + (y-7)^2 = 7^2$

$x^2 = 49 - (y-7)^2$

$x = \sqrt{49 - (y-7)^2}$

$V = lw\,\Delta y$

$= 15(2x)\Delta y$

$= 30x\,\Delta y$

$= 30\sqrt{49-(y-7)^2}\,\Delta y$

$$W = \int_0^4 \rho g (9-y)\, 30\sqrt{49-(y-7)^2}\, dy$$