152 Week In Review: Sections 6.3, 6.4, 7.1

Solutions and questions can be found at the link:
https://www.math.tamu.edu/~kahlig/152WIR.html

Shell slice needs to be parallel to axis of Rotation

1. Set up the integral(s) that would find the volume of the solid obtained by rotating the region bounded the curves

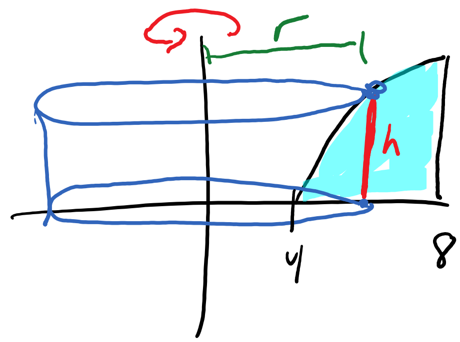
$y = \sqrt{x-4}$ $\quad x$–axis $\quad x = 8$

(a) about the y–axis.

dx Integral

$r = x$

$h = y = \sqrt{x-4}$



$A = 2\pi r h$

$$V = \int_4^8 2\pi\, x \sqrt{x-4}\, dx$$

(b) about the line $y = 5$.

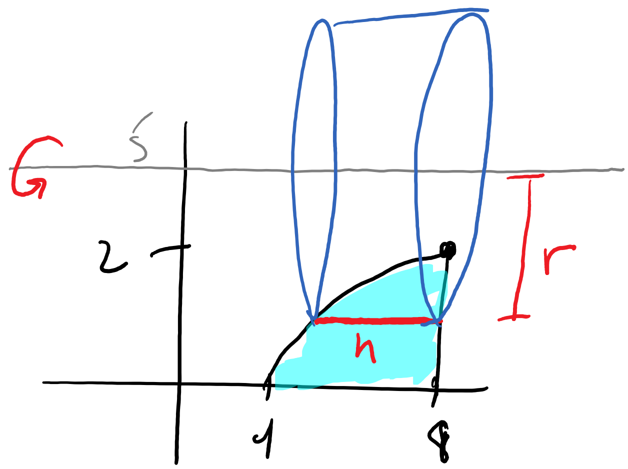
dy Integral

$r = 5 - y$

$h = 8 - (y^2 + 4)$

$= 8 - y^2 - 4$

$h = 4 - y^2$



$y = \sqrt{x-4}$

$y^2 = x - 4$

$y^2 + 4 = x$

$$V = \int_0^2 2\pi (5-y)(4-y^2)\, dy$$

2. Set up the integral(s) that would find the volume of the solid obtained by rotating the region bounded the curves about the x−axis.

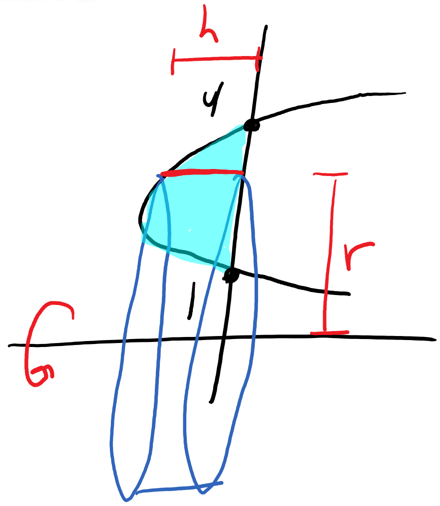
$x = y^2 - 5y + 4 \qquad y$−axis.

$X = (y-4)(y-1)$

dy Integral.

$x=0$

$y=4 \quad y=1$



$r = y = y - 0$

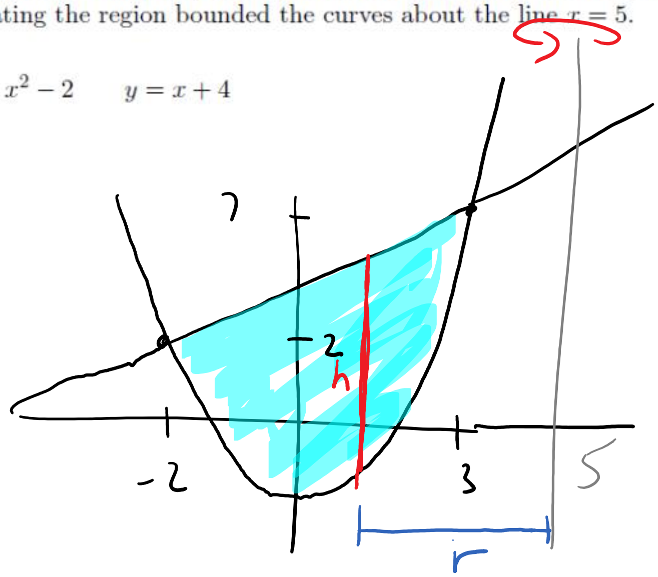
$h = 0 - (y^2 - 5y + 4)$

$= -y^2 + 5y - 4$

$V = \int_1^4 2\pi y \left(5y - y^2 - 4\right) dy$

3. Set up the integral(s) that would find the volume of the solid obtained by rotating the region bounded the curves about the line $x = 5$.

$y = x^2 - 2 \qquad y = x + 4$



$x^2 - 2 = x + 4$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3 \quad x = -2$

dx Integral

$r = 5 - x$

$h = x + 4 - (x^2 - 2)$

$= x + 4 - x^2 + 2$

$= x - x^2 + 6$

$$V = \int_{-2}^{3} 2\pi (5 - x)(x - x^2 + 6)\,dx$$

4. A particle is moved along the x−axis by a force, in newtons, of $f(x) = 3x^2 + 2x$ at a point that is x meters from the origin. Find the work done moving the particle from $x = 2$ to $x = 5$.

$$W = \int_2^5 3x^2 + 2x \, dx = x^3 + x^2 \Big|_2^5$$

$$= 125 + 25 - (8 + 4)$$

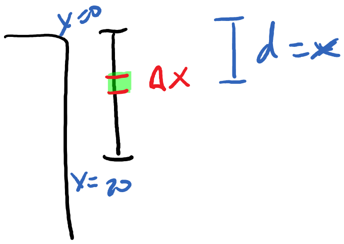
$$= 138 \text{ Nm} = 138 \text{ J}$$

5. A 20 foot rope that weighs 120 pounds is hanging over a cliff and at the end of the rope is a person that weighs 160 pounds. Find the work required to pull 10 feet of the rope to the top of the cliff.

Rope

$$\frac{120 \text{ lbs}}{20 \text{ ft}} = 6 \text{ lbs/ft}$$

method 1



$d = x$

$F = 6\,\Delta x$

$W = F \cdot d = 6x\,\Delta x$

for the green slice.

Rope pulled up. (first 10 ft)

$$W = \int_0^{10} 6x\,dx$$

Rest of the work

Rest of the rope +weight

$$\text{Answer} = \int_0^{10} 6x\,dx + \underbrace{\left[10\text{ ft}\,(6\text{ lb/ft}) + 160\right]}_{\#\text{ of lbs still over the edge}} \cdot \underbrace{10}_{\text{feet}}$$

how far this part moved.

$$= 2500 \text{ ft} \cdot \text{lbs.}$$

Page 5 method 2

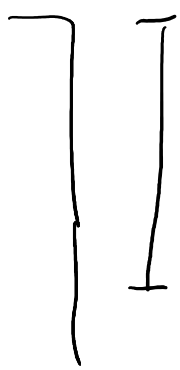
5. A 20 foot rope that weighs 120 pounds is hanging over a cliff and at the end of the rope is a person that weighs 160 pounds. Find the work required to pull 10 feet of the rope to the top of the cliff.

$x =$ amt pulled up.

Rope

$$\frac{120 \text{ lbs}}{20 \text{ ft}} = 6 \text{ lbs/ft}$$

method 2



lbs. still hanging.

$x=0$ $\quad 120 + 160 = 280$ lbs.

$x=1$ $\quad 114 + 160 = 274$ lbs

$x=2$ $\quad 108 + 160 = 268$ lbs

$\vdots$

$$F = 120 + 160 - 6x = 280 - 6x$$

Non-constant force

$$W = \int_0^{10} 280 - 6x \, dx = \ldots = 2500 \text{ ft lbs}$$

6. A 15 meter rope weighs 30 kg and is hanging off a 30 meter tall building. Find the work to bring the entire rope to the top of the building.

$$\frac{30\text{ kg}}{15\text{ m}} = 2\text{ kg/m}$$

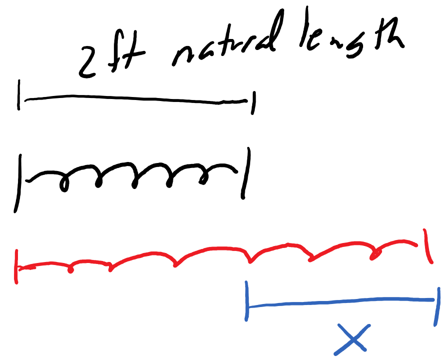
$$F = 30\text{ kg}(9.8) - 2(9.8)x$$

x = # of meters pulled up.

$$= 294 - 19.6x$$

$$W = \int_0^{15} 294 - 19.6x \, dx = \cdots = 2205\text{ J}$$

7. A spring has a natural length of 2 feet. If a force of 24 pounds is required to hold the spring to a length of 6 feet, find the work done to stretch the spring from 3 feet to 5 feet



$f = kx$

Length past the natural length

$f = kx$

$24\,lbs = k(6 - 2)$

$24 = k(4)$

$k = 6$

$f = 6x$

natural length

$$W = \int_{3-2}^{5-2} 6x \, dx = \int_1^3 6x \, dx = 3x^2 \Big|_1^3 = 27 - 3$$

$W = \underline{24 \text{ ft} \cdot \text{lbs}}$

work.

8. Suppose a spring has a natural length of 3 ft and it takes 10 ft-lb to stretch a spring from 5 ft to 8 ft.

(a) How much work is required to stretch the spring from 4 ft to 7 ft?

$$W = \int_a^b kx\,dx$$

$$10 = \int_{5-3}^{8-3} kx\,dx \rightarrow 10 = \int_2^5 kx\,dx = \left.\frac{kx^2}{2}\right|_2^5$$

$$10 = \frac{k\,25}{2} - \frac{k \cdot 4}{2}$$

mult by 2

$$20 = 25k - 4k$$

$$20 = 21k$$

$$k = \frac{20}{21}$$

force

$$f = \frac{20}{21}x$$

$$W = \int_{4-3}^{7-3} \frac{20}{21}x\,dx = \int_1^4 \frac{20}{21}x\,dx = \left.\frac{10}{21}x^2\right|_1^4$$

$$= \frac{10}{21}(16) - \frac{10}{21}(1) = \frac{50}{7}\text{ ft}\cdot\text{lbs}$$

(b) How far beyond its natural length would a force of 3 lb keep the spring stretched?

$$f = kx \qquad f = \frac{20}{21}x$$

$$3 = \frac{20}{21}x$$

$$3 \cdot \frac{21}{20} = x$$

$$x = \frac{63}{20}\text{ ft}$$

$= 3.15 ft$

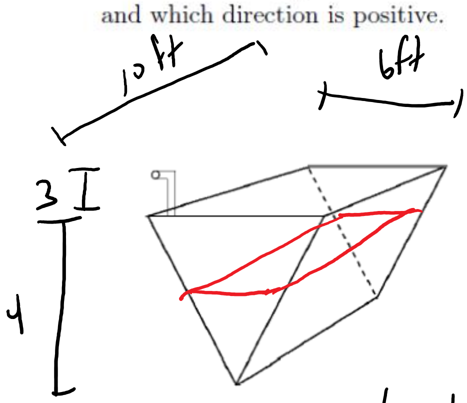
beyond natural length.

9. A tank, whose ends are isosceles triangles, has the shape as shown below. The tank is 4 feet tall (not including the spout) and is 6 feet across at the top. The tank has a 3 foot spout and has a length of 10 feet. The depth of the water in the tank is 2 feet. Use the fact that water weighs 62.5 lb/ft^3 $= \rho g$

$W = F \cdot d$

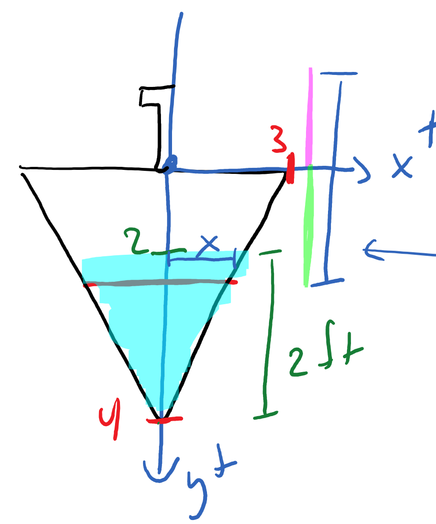
$F = \rho g V$

Set up an integral that will compute the work required to pump all of the water out of the spout. Indicate on the picture where you are placing the axis and which direction is positive.



slice is a rectangle.

$w = 2x$



$d = y + 3$

$$V = l w \, \Delta y$$
$$= 10(2x)\, \Delta y$$
$$= 20x \, \Delta y$$

$$V = 20\left(\tfrac{3}{4}\right)(4-y)\,\Delta y$$

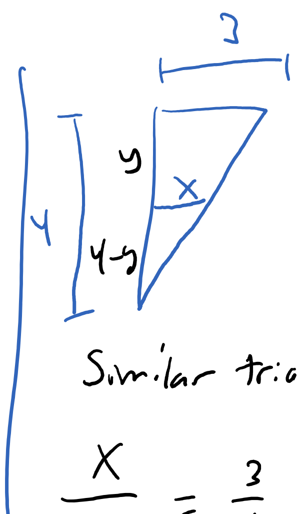
$(0,4) \quad (3,0)$

equation of line

$$m = \frac{4-0}{0-3} = -\frac{4}{3}$$

$$y - 4 = -\frac{4}{3}(x-0)$$

$$-\frac{3}{4}(y-4) = x$$



Similar triangles

$$\frac{X}{4-y} = \frac{3}{4}$$

$$X = \frac{3}{4}(4-y)$$

$$W = \int_2^4 \rho g \, 20\left(\tfrac{3}{4}\right)(4-y)(y+3)\, dy$$

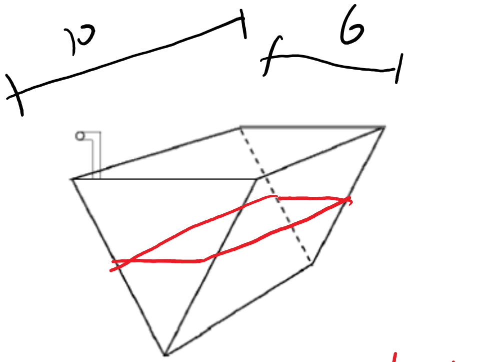
Page 9 method 2

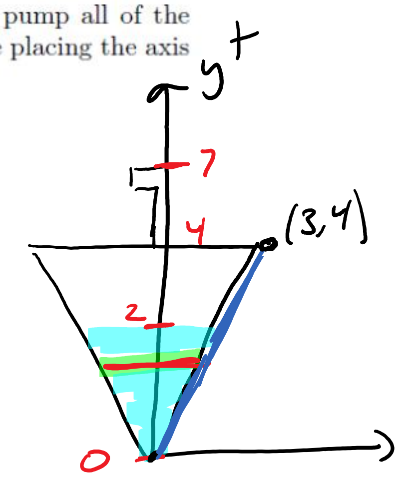
9. A tank, whose ends are isosceles triangles, has the shape as shown below. The tank is 4 feet tall (not including the spout) and is 6 feet across at the top. The tank has a 3 foot spout and has a length of 10 feet. The depth of the water in the tank is 2 feet. Use the fact that water weighs 62.5 lb/ft^3

Set up an integral that will compute the work required to pump all of the water out of the spout. Indicate on the picture where you are placing the axis and which direction is positive.

$w = F \cdot d$

$F = \rho g V$





$w = 2x$

$d = 7 - y$

slice is a Rectangle

$V = l w \, \Delta y$

$= 10(2x)\Delta y$

$= 20x \, \Delta y$

$V = 20\left(\frac{3}{4}y\right)\Delta y$

eq. of the line

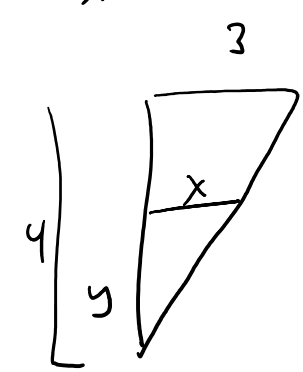
$(0,0)\ (3,4)$

$m = \frac{4-0}{3-0} = \frac{4}{3}$

$y - 0 = \frac{4}{3}(x-0)$

$y = \frac{4}{3}x$

$x = \frac{3}{4}y$



$\frac{x}{y} = \frac{3}{4}$

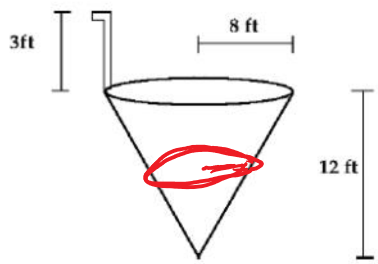
$x = \frac{3}{4}y$

$$w = \int_0^2 \rho g \, 20\left(\frac{3}{4}y\right)(7-y)\,dy$$

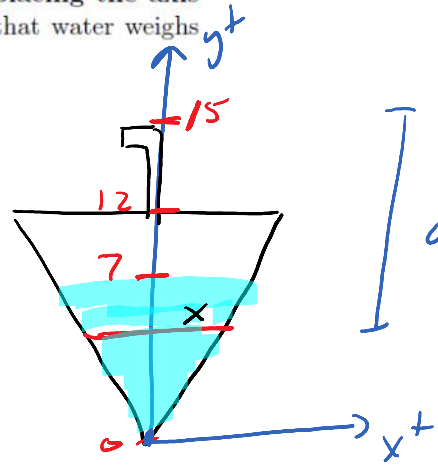
10. The conical tank shown below is 12 feet tall (not including the spout), has a 8 foot radius at the top, and has a 3 foot spout. The tank is filled with water to a depth of 7 feet.

Set up (but do not evaluate) an integral that will compute the work required to pump all the water out of the spout.

Be sure to indicate on the picture where you are placing the axis and the direction of the positive axis. Use the fact that water weighs 62.5 lb/ft^3 $= \rho g$



Slice is a circle

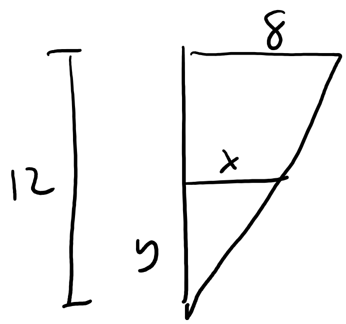


$d = 15 - y$

$V = \pi r^2 \Delta y$

$= \pi x^2 \Delta y$

$V = \pi \left(\frac{2}{3} y\right)^2 \Delta y$



Similar Triangles

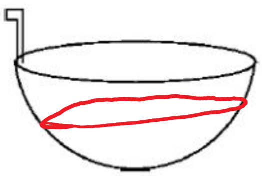
$\frac{x}{y} = \frac{8}{12}$

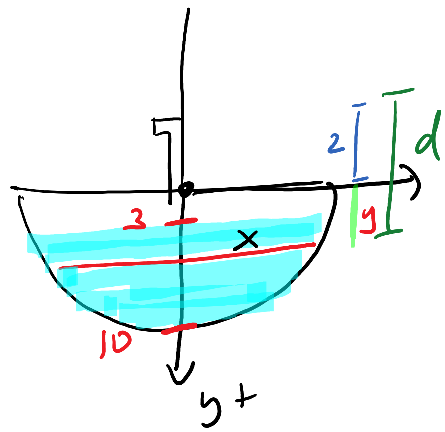
$x = \frac{2}{3} y$

$$W = \int_0^7 \rho g \, \pi \left(\frac{2}{3} y\right)^2 (15 - y) \, dy$$

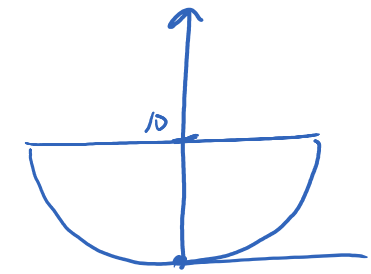
11. A Hemispherical tank has the shape shown below. The tank has a radius of 10 meters with a 2 meter spout at the ~~tope~~ top of the tank. The tank is filled with water to a depth of 7 meters. The weight density of water is $\rho g = 9800 \text{ N/m}^3$.

Set up an integral that will compute the work required to pump all of the water out of the spout. Indicate on the picture where you are placing the axis and which direction is positive.





$$(x-h)^2+(y-k)^2 = r^2$$



Slice is a circle

$$V = \pi r^2 \Delta y$$
$$= \pi x^2 \Delta y$$
$$V = \pi(100-y^2)\Delta y$$

$$d = y+2$$

$$x^2+y^2 = (10)^2$$
$$x^2+y^2 = 100$$
$$x^2 = 100-y^2$$

$$W = \int_3^{10} \rho g \pi (100-y^2)(y+2)\, dy$$