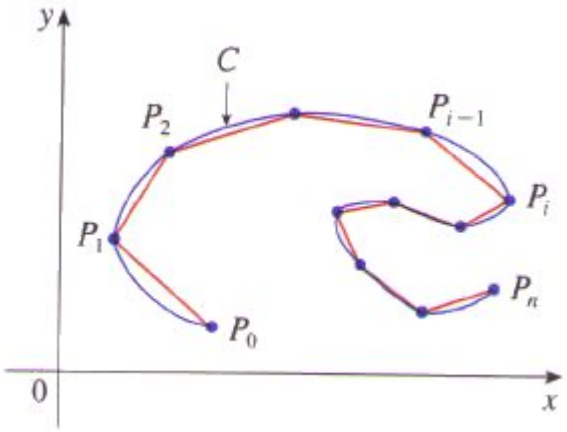
Section 10.2: Calculus with Parametric Functions.

Arc Length

Suppose that C is a smooth curve defined by $x = f(t)$ and $y = g(t)$ for $[a, b]$.
Let $\{P_i\}$ be a set of points on the curve that partition of the interval $[a, b]$ such that Δt is equal for each subinterval.



Then the length of the curve(arc length) is given by $L = \lim_{n\to\infty} \sum_{i=1}^{n} |P_{i-1}P_i| = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta s_i$

$\Delta s_i = |P_{i-1}P_i| = \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$

$\Delta s_i = \sqrt{(f'(t_i)\Delta t)^2 + (g'(t_i)\Delta t)^2}$

$\Delta s_i = \sqrt{(f'(t_i))^2 + (g'(t_i))^2}\Delta t$

Example: Find the length of the arc of the curve given by $x(t) = 3t - t^3$, $y(t) = 3t^2$ from the point $(0, 0)$ to the point $(-2, 12)$

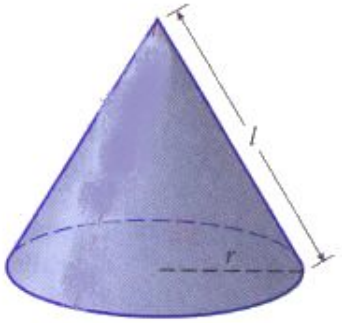
Example: Find the length of the arc of the curve $y = \frac{x^3}{6} + \frac{1}{2x}$, on the interval $1 \leq x \leq 2$.

Example: Find the length of the arc of the curve $x = 5 - \sqrt{y^3}$, from the point $(4, 1)$ to the point $(-3, 4)$

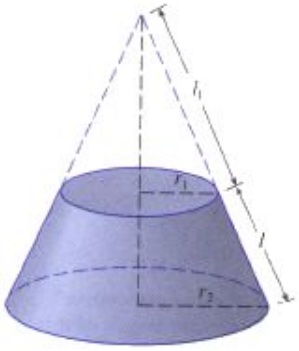
Surface Area

Rotate $y = 3$ from $x = 0$ to $x = 4$ about the x-axis. Find the surface area of the object.

Surface Area of cones.



$SA = \pi rl$



$SA = \pi(r_1 + r_2)l$

The surface area of a curve rotated about the x-axis:

The surface area of a curve rotated about the y-axis:

Example: Find the area of the surface obtained by rotating the curve $y = \sqrt{x}$ from the point $(1, 1)$ to $(4, 2)$ about the x-axis.

Example: Find the area of the surface obtained by rotating the curve $x = t$, $y = \frac{t^2}{4} - \frac{\ln(t)}{2}$ on the interval $1 \leq t \leq 4$ about the y-axis.