Section 11.8: Power Series

Definition: A **power series** centered at $x = a$ is a series of the form

$$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + ...$$

where x is a variable and c_n are constants called the coefficients of the series.

Example: Where is this power series centered?

$$\sum_{n=0}^{\infty} (2x-10)^n$$

Example: Is the following a power series?

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + ...$$

Theorem: For a given power series, $\sum_{n=0}^{\infty} c_n(x-a)^n$, there are only three possibilities for convergence.

(i) The series converges only when $x = a$

(ii) The series converges for all x.

(iii) There is a positive number R such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$.

Example: Suppose that the series $\sum_{n=0}^{\infty} c_n(x-3)^n$ converges for $x=5$ and diverges for $x=7$. For what values of x will this series converge/diverge?

Example: Find the radius and the interval of convergence for the power series.

$$0+\frac{1}{5}x+\frac{2}{5^2}x^2+\frac{3}{5^3}x^3+\frac{4}{5^4}x^4+...$$

Example: Find the radius and the interval of convergence for the power series.

$$\sum_{n=0}^{\infty} \frac{x^n}{(n+1)!}$$

Example: Find the radius and the interval of convergence for the power series.

$$\sum_{n=1}^{\infty} \frac{(x-4)^n}{\sqrt{n}}$$

Example: Find the radius and the interval of convergence for the power series.

$$\sum_{n=0}^{\infty} n!(x-1)^n$$

Example: Find the radius and the interval of convergence for the power series.

$$\sum_{n=0}^{\infty} \frac{n+1}{10^n}(3x-4)^n$$