

1. (8 points) Compute the following. If it is not possible, then write not possible.

$$(a) \quad 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & 2 \\ 1 & 0 \\ 2 & b \end{bmatrix} = \text{not possible}$$

(b) If $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} M & 7 \end{bmatrix}$, compute $BA =$

$$\begin{bmatrix} m & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 2m+7 & m & 21 \end{bmatrix}$$

2. (8 points) Solve for the variables in the problem. If it is not possible, then explain why.

$$\begin{bmatrix} 12 & 3y \\ x & 2 \end{bmatrix} - 4 \begin{bmatrix} z & 3 \\ 4 & w \end{bmatrix}^T = \begin{bmatrix} x & w \\ 16 & -18 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 3y \\ x & 2 \end{bmatrix} - 4 \begin{bmatrix} z & 4 \\ 3 & w \end{bmatrix} = \begin{bmatrix} x & w \\ 16 & -18 \end{bmatrix}$$

$$12 - 4z = x$$

$$3y - 16 = w$$

$$x - 12 = 16$$

$$2 - 4w = -18$$

$$12 - 4z = 28$$

$$3y - 16 = 5$$

$$x = 28$$

$$20 = 4w$$

$$-4z = 16$$

$$3y = 21$$

$$5 = w$$

$$z = -4$$

$$y = 7$$

3. (9 points) Give the solution to each of these linear systems represented by the matrices.

$$\left[\begin{array}{cccc|c} x & y & z & w & \\ \hline 1 & 2 & 0 & 2 & 15 \\ 0 & 0 & 1 & 5 & 9 \\ 0 & 0 & 0 & 1 & 20 \end{array} \right]$$

$$\begin{aligned} X &= 15 - 2y - 2w \\ Z &= 9 - 5w \\ w &= 20 \end{aligned}$$

$$\left[\begin{array}{ccc|c} x & y & z & \\ \hline 5 & 0 & 0 & 40 \\ 0 & 0 & 0 & 20 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

\emptyset

$$\begin{aligned} w &= 20 \\ z &= -9 \\ x &= -25 - 2y \\ y &= \text{any } \# \end{aligned}$$

$$\left[\begin{array}{ccc|c} x & y & z & \\ \hline 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 3 & 9 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\begin{aligned} X &= 7 \\ Y &= 3 \\ Z &= 3 \end{aligned}$$

4. A linear programming problem is modeled as follows:

Minimize: $f = 10x + 6y$

Constraints:

$$x + 3y \geq 6$$

$$x + 7y \geq 2$$

$$x - y \geq 2$$

$$x, y \geq 0$$

$$P = \left[\begin{array}{ccc|c} 1 & 3 & 6 \\ 1 & 7 & 2 \\ 1 & -1 & 2 \\ \hline 10 & 6 & * \end{array} \right]$$

$$P^T = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 10 \\ 3 & 7 & -1 & 6 \\ \hline 6 & 2 & 2 & * \end{array} \right]$$

(a) (5 points) Write down the formulation for the Dual problem.

$$\max f = 6u + 2v + 2w$$

$$u + v + w \leq 10$$

$$3u + 7v - w \leq 6$$

$$u, v, w \geq 0$$

(b) (3 points) Suppose we solve part (a) and get the following tableau. Find the solution to the original problem.

$$\left[\begin{array}{cccccc|c} u & v & w & s_1 & s_2 & f & \\ \hline 0 & -1 & 1 & 3/4 & -1/4 & 0 & 6 \\ 1 & 2 & 0 & 1/4 & 1/4 & 0 & 4 \\ \hline 0 & 8 & 0 & 3 & 1 & 1 & 36 \end{array} \right]$$

$$\begin{aligned} X &= 3 \\ Y &= 1 \\ f &= 36 \end{aligned}$$

5. (6 points) An economy has three sectors: R, T, and G. The input-output matrix A is shown below including the sector labels.

$$A = \begin{matrix} & \begin{matrix} R & T & G \end{matrix} \\ \begin{matrix} R \\ T \\ G \end{matrix} & \begin{bmatrix} 0.1 & 0.1 & 0.2 \\ 0.2 & 0.4 & 0.1 \\ 0.1 & 0.4 & 0.1 \end{bmatrix} \end{matrix}$$

- (a) There is a demand for 55 units of R, 60 units of T, and 175 units of G. Find the production level that would satisfy this demand.

R: 150 T: 200 G: 300

- (b) How much of the production (in part a) is consumed internally?

R: 95 T: 140 G: 125

6. (8 points) Set up the following linear programming problem. Be sure to define your variables. DO NOT SOLVE.

A real estate developer is planning to build a new apartment complex consisting of one-bedroom units and two- and three-bedroom townhouses. The complex will have at most 450 units and the number of family units (two- and three-bedroom townhouses) will be at least three times the number of one-bedroom units. Due to local ordinances, the complex can not have anymore than 90 one bedroom units. The developer believes they can rent the one-bedroom units for \$950 and the two- and three-bedroom townhouses for \$1700 and \$2400 respectively. How many units of each type will be in the complex if they want to maximize the revenue from renting the apartments?

$x = \#$ of one bedroom
 $y = \#$ of 2 bedroom
 $z = \#$ of 3 bedroom

max
obj: $R = 950x + 1700y + 2400z$

constraints:

$$x + y + z \leq 450$$

$$y + z \geq 3x$$

$$x \leq 90$$

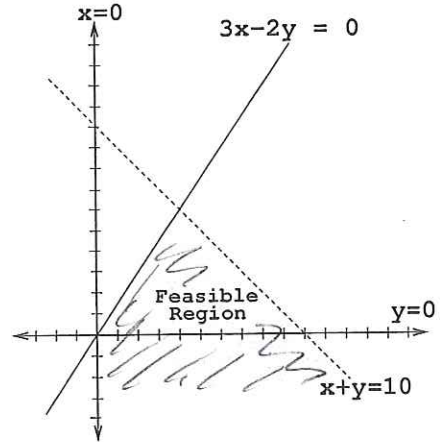
$$x, y, z \geq 0$$

7. (6 points) Write down the system of inequalities that will give the feasible region to the right.

$$x \geq 0$$

$$x + y \leq 10$$

$$3x - 2y \geq 0$$

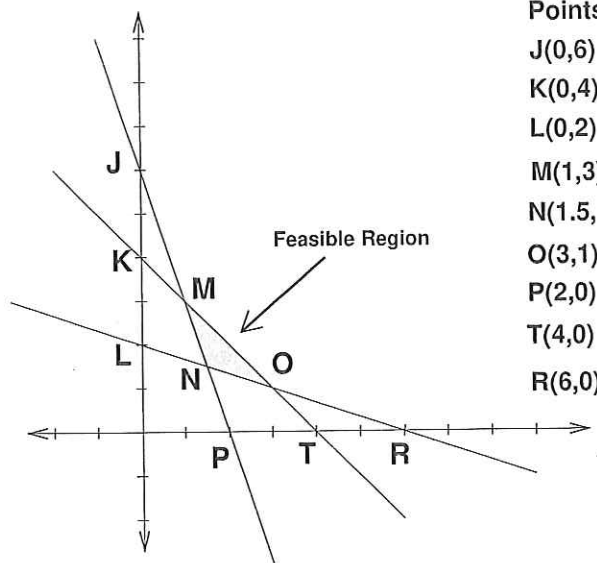


8. (5 points) Use the feasible region to answer these questions. If the answer doesn't exist, then be sure to provide work to justify your answer.

At what point(s) does the objective function $f = x + 3y$ have a minimum and what is the minimum value?

Value: 6

Location: NO



- Points
- J(0,6)
 - K(0,4)
 - L(0,2)
 - M(1,3) = 10
 - N(1.5, 1.5) = 6
 - O(3,1) = 6
 - P(2,0)
 - T(4,0)
 - R(6,0)

9. (5 points) Set up the first matrix that you would use when solving this linear programming problem.

Maximize: $P = 5x + 2y$

Constraints:

$$x + 2y = 6$$

$$x + y \geq 3$$

$$y \leq 25$$

$$x \geq 0$$

$$y \geq 0$$

$$x + 2y \leq 6$$

$$x + y \geq 3$$

x	y	s_1	s_2	s_3	s_4	P		
1	2	1	0	0	0	0	6	
-1	-2	0	1	0	0	0	-6	
-1	-1	0	0	1	0	0	-3	
0	1	0	0	0	1	0	25	
							1	0
-5	-2	0	0	0	0	1	0	

10. ²⁰~~(15)~~ points) The following matrices represent some step in solving a linear programming problem.

If the matrix is not in a final form, then tell the location of the next pivot element as was discussed during class. If there is no solution, then write no solution.

If the matrix is in final form, then give the solution. Be sure to tell if the problem has an infinite number of solutions.

(a)
$$\begin{array}{c} \rightarrow \\ \left[\begin{array}{ccccccc|c} x & y & z & s_1 & s_2 & s_3 & P & \\ 0 & 5 & 10 & 0 & 3 & 1 & 0 & 10 \\ 1 & -3 & -4 & 0 & 7 & 0 & 0 & -16 \\ 0 & 3 & -3 & 1 & -2 & 0 & 0 & 1 \\ \hline 0 & 5 & 2 & 0 & -6 & 0 & 1 & 18 \end{array} \right] \end{array}$$

↑

R1C3

(d)
$$\left[\begin{array}{ccccccc|c} x & y & z & s_1 & s_2 & s_3 & P & \\ 3 & 2 & 0 & 1 & -4 & 0 & 0 & 2 \\ 3 & 1 & 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 4 & 0 & 0 & -1 \\ \hline 0 & -3 & 0 & 0 & 2 & 0 & 1 & 7 \end{array} \right]$$

no solution

(b)
$$\left[\begin{array}{ccccccc|c} x & y & z & s_1 & s_2 & s_3 & P & \\ 0 & 5 & 0 & 0 & 3 & 1 & 0 & 11 \\ 1 & 0 & -2 & 0 & 7 & 0 & 0 & 5 \\ 0 & 3 & -3 & 1 & -2 & 0 & 0 & 3 \\ \hline 0 & -5 & 5 & 0 & -2 & 0 & 1 & 18 \end{array} \right]$$

↑

R3C2

(e)
$$\left[\begin{array}{ccccccc|c} x & y & z & s_1 & s_2 & s_3 & P & \\ 1 & 2 & 0 & 1 & 4 & 0 & 0 & 6 \\ -3 & 1 & 0 & 0 & -2 & 1 & 0 & -4 \\ 1 & 2 & 1 & 0 & -4 & 0 & 0 & 0 \\ \hline 0 & -3 & 0 & 0 & 2 & 0 & 1 & 7 \end{array} \right]$$

↑

R2C1

(c)
$$\left[\begin{array}{ccccccc|c} x & y & z & s_1 & s_2 & s_3 & P & \\ -1 & 2 & 0 & 1 & 4 & 0 & 0 & 8 \\ -3 & 1 & 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & -4 & 0 & 0 & 1 \\ \hline -5 & -3 & 0 & 0 & -2 & 0 & 1 & 7 \end{array} \right]$$

↑

no solution

11. (10 points) A confectioner makes three types of candy: Sweet Tooth, Sugar Dandy, and Dandy Delite. Each box of candy has the following requirements. The current inventory of each ingredient is also listed in the table.

	Sweet Tooth	Sugar Dandy	Dandy Delite	inventory
chocolate	3 lbs	4 lbs	5 lbs	500 lbs
nuts	1 lb	0.5 lbs	0.75 lbs	70 lbs
fruit	1 lb		1 lb	76 lbs

Each box of Sweet Tooth sells for \$4, Sugar Dandy sells for \$6, and Dandy Delite sells for \$8.

For this problem x = the number of boxes of Sweet Tooth, y = the number of boxes of Sugar Dandy, z = the number of boxes of Dandy Delite and R = revenue.

First simplex matrix:

The last simplex matrix is

$$\left[\begin{array}{ccccccc|c} x & y & z & s_1 & s_2 & s_3 & R & \\ \hline 3 & 4 & 5 & 1 & 0 & 0 & 0 & 500 \\ 1 & 0.5 & 0.75 & 0 & 1 & 0 & 0 & 70 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 76 \\ \hline -4 & -6 & -8 & 0 & 0 & 0 & 1 & 0 \end{array} \right] 76$$

$$\left[\begin{array}{ccccccc|c} x & y & z & s_1 & s_2 & s_3 & R & \\ \hline -4 & 0 & 0 & 1 & -8 & 1 & 0 & 16 \\ -5.5 & 1 & 0 & 1.5 & -10 & 0 & 0 & 50 \\ 5 & 0 & 1 & -1 & 8 & 0 & 0 & 60 \\ \hline 3 & 0 & 0 & 1 & 4 & 0 & 1 & 780 \end{array} \right]$$

- (a) How many boxes of each type of candy should be made from the available inventory in order to maximize the revenue?

*no boxes of sweet tooth
50 boxes of Sugar dandy
60 boxes of Dandy Delite.*

- (b) After maximizing the revenue, is there any inventory left over? If yes, then list the left over ingredient(s) and their amounts.

16 lbs of chocolate

- (c) Use shadow values to determine the revenue the company will make if its supply of chocolate increases to 600 lbs, its supply of nuts decreases by 10 lbs, and its supply of fruit decreases by 4 lbs.

$$100(1) + (-10)(4) + (-4)(0) + 780 = 840$$

12. (3 points) A village has two industries: A and B. Production of one unit of A requires 0.3 units of A and 0.1 unit of B. Production of one unit of B requires 0.2 units of A and 0.4 units of B. Find the input-output matrix. Label the columns and rows.

$$\begin{matrix} & A & B \\ A & \begin{bmatrix} .3 & .2 \end{bmatrix} \\ B & \begin{bmatrix} .1 & .4 \end{bmatrix} \end{matrix}$$

13. (9 points) For the word problem do the following.

- A) Set up the system of equations. Be sure to define the variables.
- B) Solve the problem using any method that was used in class.
- C) If the solution is parametric, tell what restriction can be placed on the parameter.

An exotic ranch wants to purchase some animals: Wallaby, Squirrel Monkey, and Macaw. They have decided that they have space for 50 new critters. They have \$500,000 to spend on the animals. It will cost \$7,000 for each Wallaby, \$9,000 for each Squirrel Monkey, and \$20,000 for each Macaw. How many of each type of animal can be purchased.

$x = \# \text{ of wallabys}$
 $y = \# \text{ of monkey}$
 $z = \# \text{ of Macaw}$

$$\begin{aligned} x + y + z &= 50 \\ 7000x + 9000y + 20000z &= 500000 \end{aligned}$$

rref $\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -5.5 & -25 \\ 0 & 1 & 6.5 & 75 \end{array} \right]$

$$\begin{aligned} x &= 5.5z - 25 \\ y &= 75 - 6.5z \end{aligned}$$

$$\begin{aligned} x &\geq 0 \\ 5.5z - 25 &\geq 0 \\ 5.5z &\geq 25 \\ z &\geq 4.545 \end{aligned}$$

$$\begin{aligned} x &\leq 50 \\ 5.5z - 25 &\leq 50 \\ 5.5z &\leq 75 \\ z &\leq 13.6 \end{aligned}$$

$$\begin{aligned} y &\geq 0 \\ 75 - 6.5z &\geq 0 \\ 75 &\geq 6.5z \\ 11.53 &\geq z \end{aligned}$$

$$\begin{aligned} y &\leq 50 \\ 75 - 6.5z &\leq 50 \\ 25 &\leq 6.5z \\ 3.8 &\leq z \end{aligned}$$

$$4.5 \leq z \leq 11.53$$

Integer:

$$z = 5, 6, 7, 8, 9, 10, 11$$

to make $x + y$ Integers.

$$z = 6, 8, 10$$