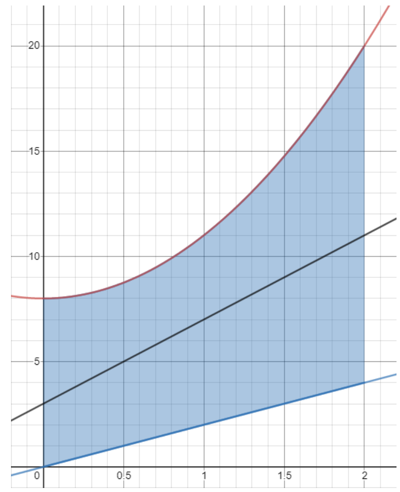
Section 6.1: Additional Problems Solutions

1. Find the value of m so that the line $y = mx + 3$ will bisect(divide the area in half) the area that is bounded by $y = 3x^2 + 8$, and $y = 2x$ on the interval $[0, 2]$. Hint: draw a picture.

The picture shows the two functions(red and blue). We want the shaded region to be divided into two regions of equal area. That is represented by the black function.
step 1: Find the area of the blue shaded region.



$$\int_0^2 3x^2 + 8 - (2x)dx = (x^3 + 8x - x^2)\Big|_0^2$$

$$= 8 + 16 - 4 - (0) = 20$$

Setp 2: Now solve for m.

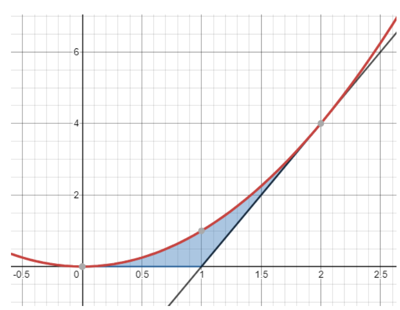
$$10 = \int_0^2 mx + 3 - 2xdx = (\frac{mx^2}{2} + 3x - x^2)\Big|_0^2$$

$10 = 2m + 6 - 4 = 2m + 2$

$m = 4$

2. Sketch the region that is bounded by the curve $y = x^2$, the tangetn line to this curve at $x = 2$, the $x-$axis, and the $y-$axis. Compute the are of this region.

Step 1: Find the equation of the tangent line.



$y' = 2x$ so the slope of the tangent line at $x = 2$ is
$m = 2(2) = 4$.
the y-coordinate at $x = 2$ is $y = 2^2 = 4$.
tangent line: $y - 4 = 4(x - 2)$ or $y = 4x - 4$
The graph of the region is to the right. We see that the tangent line crosses the x-axis at $x = 1$.
This region can be evaluated with a single dy integral or two dx integrals.

<u>dy integral</u>

solve both functions for $x =$ to get $x = \sqrt{y}$ and $x = \frac{y+4}{4}$

$$\int_0^4 \frac{y+4}{4} - \sqrt{y}\, dy = \ldots = \frac{2}{3}$$

<u>dx integrals</u>

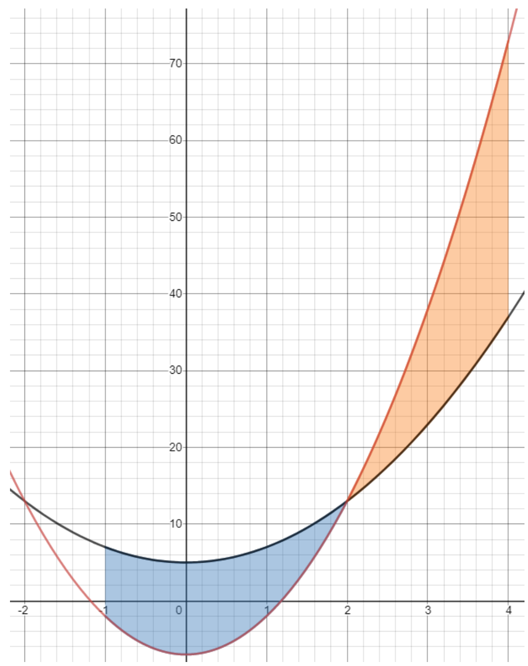
$$\int_0^1 x^2 dx + \int_1^2 x^2 - (4x - 4)\, dx = \ldots = \frac{2}{3}$$

3. Find the area bounded by these curves on the interval from $x = -1$ to $x = 4$.

   $y = 2x^2 + 5$ (black function)

   $y = 5x^2 - 7$ (red function)

First graph the two functions on the interval $[-1, 4]$.



Notice that there are two regions and the top function in the blue region is the bottom function in the orange region.

This means we have to use two integrals.

now find the x-values of where the functions intersect.

$2x^5 = 5x^2 - 7$

$12 = 3x^2$

$x^2 = 4$

$x = \pm 2$

$$\int_{-1}^{2} 2x^2 + 5 - (5x^2 - 7)\ dx + \int_{2}^{4} 5x^2 - 7 - (2x^2 + 5)\ dx$$

$$\int_{-1}^{2} -3x^2 + 12x\ dx + \int_{2}^{4} 3x^2 - 12x\ dx = \ldots = 59$$