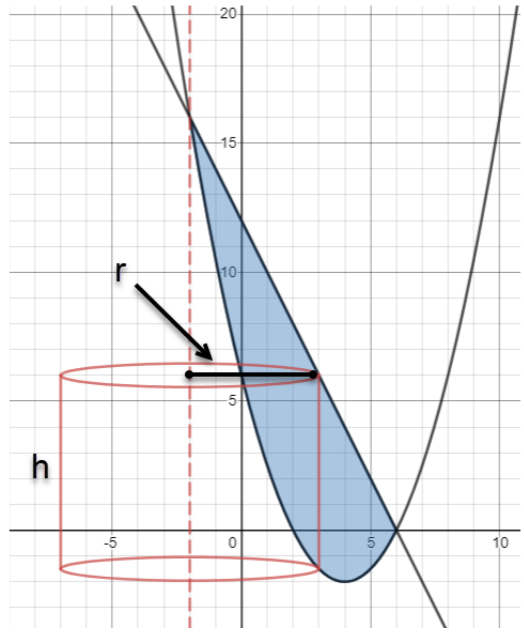
Section 6.3: Additional Problems Solutions

1) Set up the integral(s) that would give the volume of the solid obtained by rotating the region bounded by the given curves around the $x = -2$.



$y = -2x + 12$
$y = 0.5x^2 - 4x + 6$

Step 1: sketch the graph of the region rotated and find the intersection values of the curves.

$-2x + 12 = 0.5x^2 - 4x + 6$
$0 = 0.5x^2 - 2x - 6$
$0 = x^2 - 4x - 12$
$0 = (x - 6)(x + 2)$
$x = 6$ ot $x = -2$

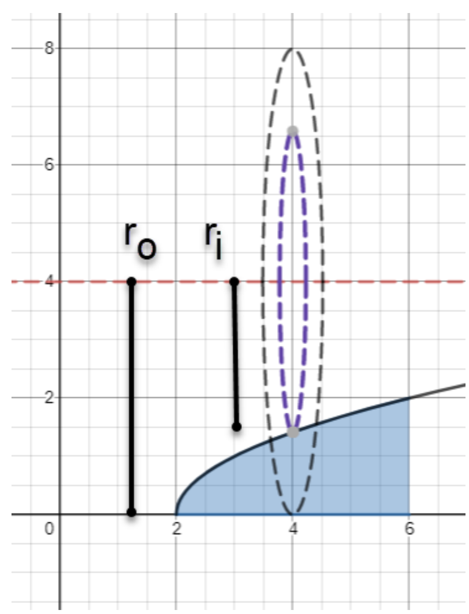
This is a dx integral since the slice is perpendicular to the $x-$axis.

Step 2: Now find the formula for the radius and the height.

$h = -2x + 12 - (0.5x^2 - 4x + 6 = -0.5x^2 + 2x + 6$ (top - bottom)
$r = x - (-2) = x + 2$ (right - left)
Step 3: Setup the integral.

$$V = \int_{-2}^{6} 2\pi r h dx = \int_{-2}^{6} 2\pi(x + 2)(-0.5x^2 + 2x + 6)\ dx = \cdots = \frac{1024\pi}{3}$$

2) Set up the integral(s), using both methods washer/disk and cylindrical shells, that would give the volume of the solid obtained by rotating the region bounded by the following around $y = 4$. Compute this volume using the method that seems the easiest.



$y = \sqrt{x - 2}$
$y = 0$
$x = 6$

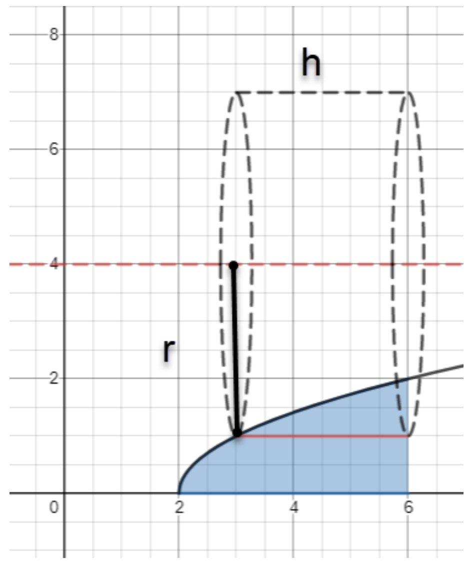
Washer Method:

This is a dx integral since the slice is perpendicular to the $x-$axis.
outer radius: $r_o = 4$
inner radius: $r_i = 4 - \sqrt{x - 2}$ (top - bottom)

Integral: $\displaystyle\int_{2}^{6} \pi \left[4^2 - \left(4 - \sqrt{x-2}\right)^2\right]\ dx$

Cylindrical shell method:

This is a dy integral since the slice(solid red line segment) is perpendicular to the y-axis.
Need to solve the square root function for $x =$.
$y = \sqrt{x-2}$ becomes $x = y^2 + 2$



$r = 4 - y$ (top - bottom)

$h = 6 - (y^2 + 2) = 4 - y^2$ (right - left)

Integral: $\int_0^2 2\pi(4-y)(4-y^2)\ dy$

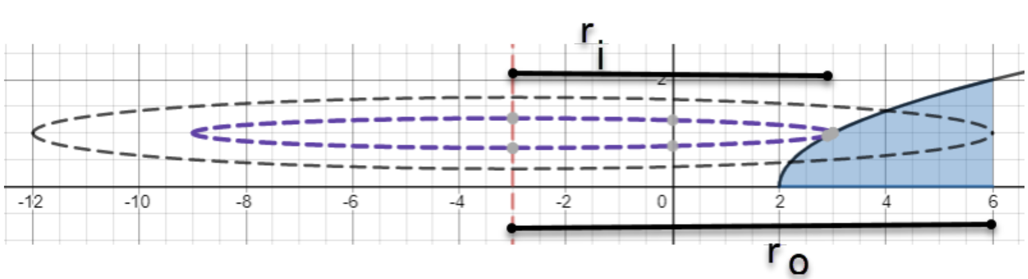
Of the two integrals, the shell method is the easier one to compute.

$$\int_0^2 2\pi(4-y)(4-y^2)\ dy = 2\pi \int_0^2 16 - 4y - 4y^2 + y^3 dy$$

$$= 2\pi \left[16y - 2y^2 - \frac{4y^3}{3} + \frac{y^4}{4}\right]_0^2 = 2\pi \left[32 - 8 - \frac{32}{3} + 4 - (0)\right] = \cdots = \frac{104\pi}{3}$$

3) Set up the integral(s), using both methods washer/disk and cylindrical shells, that would give the volume of the solid obtained by rotating the region bounded by the following around $x = -3$. Compute this volume using the method that seems the easiest.

$y = \sqrt{x-2}$
$y = 0$
$x = 6$
Washer Method:



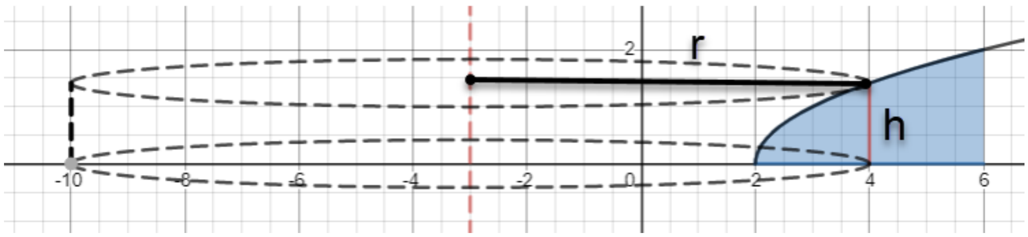
This is a dy integral since the slice is perpendicular to the x-axis.

Need to solve the square root function for $x =$.
$y = \sqrt{x-2}$ becomes $x = y^2 + 2$

outer radius: $r_o = 6 - (-3) = 9$ (right - left)
inner radius: $r_i = x - (-3) = x + 3 = y^2 + 2 + 3 = y^2 + 5$ (right - left)

Inegral: $\int_0^2 \pi \left[9^2 - (y^2+5)^2\right]\ dy$

Cylindrical shell method:



This is a dx integral since the slice(solid red line segment) is perpendicular to the x-axis.
$r = x - (-3) = x + 3$ (right - left)

$h = y - 0 = \sqrt{x-2}$ (top-bottom)

Inegral: $\int_{2}^{6} 2\pi(x+3)\sqrt{x-2}\ dx$

Of the two methods, the washer is the easiest to integrate.

$V = \int_{0}^{2} \pi \left[9^2 - (y^2+5)^2\right]\ dy = \cdots = \frac{1184\pi}{15}$