Section 7.1: Additional Problems Solutions

1. $\int \arctan(2x)\ dx$

$$\int \arctan(2x)\ dx = x\arctan(2x) - \int \frac{2x}{1+4x^2}\ dx$$

Derivative		Integral
$\arctan(2x)$	+	1
$\frac{2}{1+(2x)^2}$	$-\int$	x

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now use a u-sub with $u = 4x^2+1$ for $\int \frac{2x}{1+4x^2}\ dx$

$du = 8xdx$ and $\frac{1}{8}du = xdx$ so we get

$$\int \frac{2x}{1+4x^2}\ dx = \int \frac{1}{4}\cdot\frac{1}{u}\ du = \frac{1}{4}\ln|u| = \frac{1}{4}\ln|4x^2+1|$$

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thus $\int \arctan(2x)\ dx = x\arctan(2x) - \frac{1}{4}\ln(4x^2+1) + C$

2. $\int x\arctan(x)\ dx$

$$\int x\arctan(x)\ dx = \frac{x^2}{2}\arctan(x) - \frac{1}{2}\int \frac{x^2}{(1+x^2)}\ dx$$

Derivative		Integral
$\arctan(x)$	+	x
$\frac{1}{1+x^2}$	$-\int$	$\frac{x^2}{2}$

Now for the integral either do long division or the "trick" of adding zero.

$$\frac{1}{2}\int \frac{x^2}{(1+x^2)}\ dx = \frac{1}{2}\int \frac{x^2+1-1}{x^2+1}\ dx = \frac{1}{2}\int \frac{x^2+1}{x^2+1} - \frac{1}{x^1+1}\ dx$$

$$\frac{1}{2}\int \frac{x^2}{(1+x^2)}\ dx = \frac{1}{2}\int 1 - \frac{1}{x^1+1}\ dx = \frac{1}{2}(x - \arctan(x))$$

Thus we get

$$\int x\arctan(x)\ dx = \frac{x^2}{2}\arctan(x) - \frac{1}{2}(x - \arctan(x)) + C$$

or

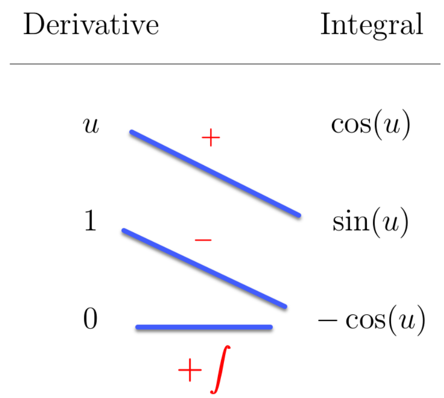
$$\int x\arctan(x)\ dx = \frac{x^2}{2}\arctan(x) - \frac{x}{2} + \frac{1}{2}\arctan(x) + C$$

3. $\int 12x^5\cos(2x^3)\ dx$ hint: u-sub

First do a u-sub with $u = 2x^3$ and $du = 6x^2 dx$

$$\int 12x^5\cos(2x^3)\ dx = \int 2x^3 \cdot 6x^2\cos(2x^3)\ dx = \int u\cos(u)du$$

Now integration by parts gives.



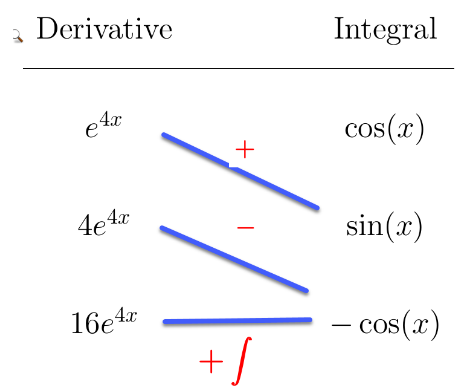
$$\int u\cos(u)du = u\sin(u) + \cos(u) + \int 0\ du$$
$$\int u\cos(u)du = u\sin(u) + \cos(u) + C$$

now go back to the variable x.

$$\int 12x^5\cos(2x^3)\ dx = 2x^3\sin(2x^3) + \cos(2x^3) + C$$

4. $\int e^{4x}\cos(x)\ dx$ hint:loop

Since this is a loop, I can put the e^{4x} in the derivative or the integral column.



notice that the last line looks like a constant multiple of the first line. This is how I know that this solution involves a loop.

$$\int e^{4x}\cos(x)\ dx = e^{4x}\sin(x) + 4e^{4x}\cos(x) - \int 16e^{4x}\cos(x)\ dx$$
$$\int e^{4x}\cos(x)\ dx = e^{4x}\sin(x) + 4e^{4x}\cos(x) - 16\int e^{4x}\cos(x)\ dx$$
$$17\int e^{4x}\cos(x)\ dx = e^{4x}\sin(x) + 4e^{4x}\cos(x)$$
$$\int e^{4x}\cos(x)\ dx = \frac{1}{17}\left[e^{4x}\sin(x) + 4e^{4x}\cos(x)\right]$$

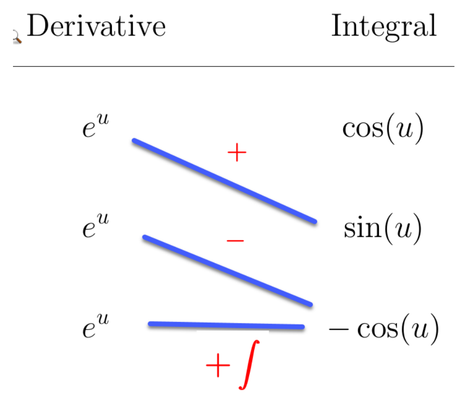
Answer: $\int e^{4x}\cos(x)\ dx = \frac{1}{17}\left[e^{4x}\sin(x) + 4e^{4x}\cos(x)\right] + C$

5. $\int \cos(\ln(x))\ dx$ hint:u-sub then loop

let $u = \ln(x)$ then $du = \frac{1}{x}dx$ or $xdu = dx$. Since $x = e^u$ we get that $e^u du = dx$.

$$\int \cos(\ln(x))\ dx = \int e^u \cos(u)\ du$$

Now do integration by parts.



$$\int e^u \cos(u) du = e^u \sin(u) + e^u \cos(u) - \int e^u \cos(u)\ du$$

$$2\int e^u \cos(u) du = e^u \sin(u) + e^u \cos(u)$$

$$\int e^u \cos(u) du = \frac{1}{2}\left(e^u \sin(u) + e^u \cos(u)\right) + C$$

Thus

$$\int \cos(\ln(x))\ dx = \frac{1}{2}\left(e^{\ln(x)} \sin(\ln(x)) + e^{\ln(x)} \cos(\ln(x))\right) + C$$

or

$$\int \cos(\ln(x))\ dx = \frac{1}{2}\left(x \sin(\ln(x)) + x \cos(\ln(x))\right) + C$$