10.2 #1

$x' = 2t \qquad y' = 3t^2$

$$ds = \sqrt{(x')^2 + (y')^2}\, dt$$

$$= \sqrt{(2t)^2 + (3t^2)^2}\, dt$$

$$= \sqrt{4t^2 + 9t^4}\, dt$$

$$L = \int_1^2 ds = \int_1^2 \sqrt{4t^2 + 9t^4}\, dt = \int_1^2 \sqrt{t^2(4 + 9t^2)}\, dt$$

$$= \int_1^2 t\sqrt{4 + 9t^2}\, dt$$

$u = 4 + 9t^2$

$du = 18t\, dt$

$\frac{1}{18} du = t\, dt$

$$= \int_{t=1}^{t=2} \frac{1}{18}\sqrt{u}\, du$$

$$= \frac{1}{18} u^{3/2} \cdot \frac{2}{3} \Big|_{t=1}^{t=2} = \frac{1}{27}\left(4 + 9t^2\right)^{3/2} \Big|_1^2$$

$$= \frac{1}{27}\left[40^{3/2} - 13^{3/2}\right]$$