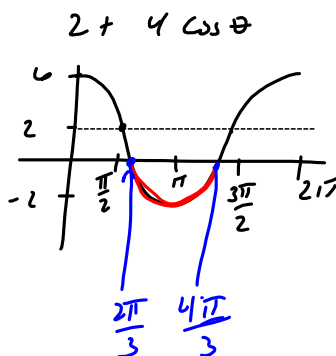
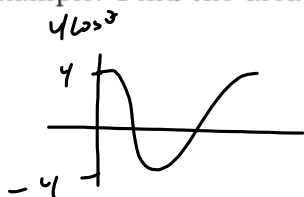


$$\cos \frac{\pi}{3} = \frac{1}{2}$$

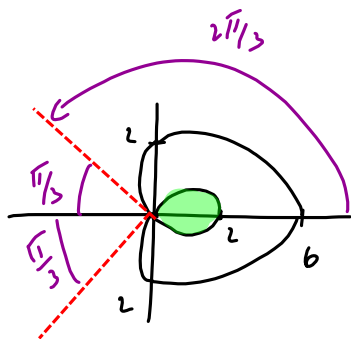
Example: Find the area of the inner loop of $r = 2 + 4 \cos \theta$



$$2 + 4 \cos \theta = 0$$

$$4 \cos \theta = -2$$

$$\cos \theta = -\frac{1}{2}$$



$$\text{Area} = \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \frac{1}{2} r^2 d\theta$$

$$= 2 \int_{\frac{2\pi}{3}}^{\pi} \frac{1}{2} (2 + 4 \cos \theta)^2 d\theta = \int_{\frac{2\pi}{3}}^{\pi} (2 + 4 \cos \theta)^2 d\theta$$

$$= \int_{\frac{2\pi}{3}}^{\pi} 4 + 16 \cos \theta + 16 \cos^2 \theta d\theta$$

$$= \int_{\frac{2\pi}{3}}^{\pi} 4 + 16 \cos \theta + 16 \cdot \frac{1}{2} [1 + \cos 2\theta] d\theta$$

$$= 4\theta + 16 \sin \theta + 8 \left[\theta + \frac{1}{2} \sin 2\theta \right] \Big|_{\frac{2\pi}{3}}^{\pi}$$

$$= \dots = 4\pi - 6\sqrt{3}$$