1) Find a MacLaurin series for these functions.

A) $f(x) = x^3 \sin(2x)$

We know that the MacLaurin series for sin(x) is the folowing.

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots \qquad R = \infty$$

thus sin(2x) = $\displaystyle\sum_{n=0} \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!} = \sum_{n=0} \frac{(-1)^n 2^{2n+1} x^{2n+1}}{(2n+1)!}$

$$x^3 \sin(2x) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+4}}{(2n+1)!}$$

1) Find a MacLaurin series for these functions.

B) $f(x) = \cos^2(x)$

We know the MacLaurin series for cos(x) is

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots \qquad R = \infty$$

however we want the series cos 2(x). While taking the series for cos(x) and squaring it, foiling out two infinite polynomials, is doable it is not a recomended task. Thus we consider the trig identites for cos 2(x).

$$\cos^2(x) = \frac{1}{2}\left(1 + \cos(2x)\right) = \frac{1}{2} + \frac{1}{2}\cos(2x)$$

$$= \frac{1}{2} + \frac{1}{2}\sum_{n=0} \frac{(-1)^n (2x)^{2n}}{(2n)!}\Bigg)$$

$$= \frac{1}{2} + \frac{1}{2}\sum_{n=0} \frac{(-1)^n 2^{2n} x^{2n}}{(2n)!}\Bigg)$$

$$= \frac{1}{2} + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n-1} x^{2n}}{(2n)!}$$

Now lets see if we can write all of the terms as a single summation. The first step is to expand out the series and then group terms.

$$= \frac{1}{2} + \left(\frac{1}{2} - \frac{2x^2}{2!} + \frac{2^3 x^4}{4!} - \frac{2^5 x^6}{6!} + \ldots \right)$$

$$= 1 - \frac{2x^2}{2!} + \frac{2^3 x^4}{4!} - \frac{2^5 x^6}{6!} + \ldots$$

Now looking at the series, in expanded form, I can not find a patter that will include all of the terms. I can see a pattern for the for everything but the first term. This gives the series below.

$$= 1 + \sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n-1} x^{2n}}{(2n)!}$$

In reality, both of these series are good answers for this question.

$$\cos^2(x) = \frac{1}{2} + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n-1} x^{2n}}{(2n)!}$$

$$\cos^2(x) = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n-1} x^{2n}}{(2n)!}$$

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1) Find a MacLaurin series for these functions.

C) $f(x) = \ln(3 + x)$

$$f'(x) = \frac{1}{3+x} = \frac{1}{3}\,\frac{1}{1+\frac{x}{3}} = \frac{1}{3}\,\frac{1}{\left(1 - \frac{-x}{3}\right)}$$

$$= \frac{1}{3}\sum_{n=0}\left(\frac{-x}{3}\right)^n = \frac{1}{3}\sum_{n=0}\frac{(-1)^n x^n}{3^n}$$

$$f'(x) = \sum_{n=0}\frac{(-1)^n x^n}{3^{n+1}}$$

$$f(x) = C + \int f'(x)\,dx = C + \int \sum_{n=0}\frac{(-1)^n x^n}{3^{n+1}}$$

$$= C + \sum_{n=0}\frac{(-1)^n x^{n+1}}{(n+1)\,3^{n+1}}$$

now to solve for the constant. Plug in x=0 to get f(0) = ln(3).

$$f(0) = \ln(3+0) = C + \sum 0$$

$$\ln(3) = C$$

$$f(x) = \ln(3) + \sum_{n=0}\frac{(-1)^n x^{n+1}}{(n+1)\,3^{n+1}}$$