4) Find the first three nonzero terms in the Maclaurin series for $y = \sec(x)$

$$\sec(x) = \frac{1}{\cos(x)}.$$

We want to use the MacLaurin series for cos(x) and perform long division. Note we only want the first three non-sero terms not the general formula for the series.

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$1 + \frac{x^2}{2} + \frac{5x^4}{24} + \dots$$

$$1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \dots \;\Big)\; 1 + 0x^2 + 0x^4 + 0x^6 + \dots$$

$$-\left(1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots\right)$$

$$\frac{x^2}{2} - \frac{x^4}{24} + \frac{x^6}{720} - \dots$$

$$-\left(\frac{x^2}{2} - \frac{x^4}{4} + \frac{x^6}{48} - \dots\right)$$

$$\frac{5x^4}{24} - \frac{7x^6}{360} + \dots$$

scratch work for the long division

$$-\frac{x^4}{24} - \frac{-x^4}{4} = -\frac{x^4}{24} + \frac{x^4}{4} = -\frac{x^4}{24} + \frac{6x^4}{24} = \frac{5x^4}{24}$$

$$\frac{x^6}{720} - \frac{x^6}{48} = \frac{x^6}{720} - \frac{15x^6}{720} = \frac{-14x^6}{720} = \frac{-7x^6}{360}$$

Answer.

$$\sec(x) \approx 1 + \frac{x^2}{2} + \frac{5x^4}{24}$$