

5) Find the 20th derivative at $x = 2$ for $f(x) = \sum_{n=0}^{\infty} \frac{2^n}{n+5} (x-2)^n$

The general formula for a Taylor series center at $x=a$ is

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n \quad \text{with } c_n = \frac{f^{(n)}(a)}{n!}$$

thus we know $c_n = \frac{2^n}{n+5} = \frac{f^{(n)}(2)}{n!}$

now we have a formula for the n -th derivative of $f(x)$ evaluated at $x=2$.

$$f^{(n)}(2) = \frac{2^n \cdot n!}{n+5}$$

thus.

$$f^{(20)}(2) = \frac{2^{20} \cdot 20!}{25}$$