

(2) Use geometric series to express $3.\overline{153}$ as a ratio of integers.

$3.153535353\dots$ can be written as

$$3.1 + .053 + .00053 + .0000053 + \dots$$

$$3.1 + \frac{53}{1000} + \frac{53}{100000} + \frac{53}{10000000} + \dots$$

$$3.1 + \frac{53}{1000} + \frac{53}{1000} \cdot \frac{1}{100} + \frac{53}{1000} \cdot \frac{1}{10000} + \dots$$

$$3.1 + \frac{53}{1000} + \frac{53}{1000} \left(\frac{1}{100}\right) + \frac{53}{1000} \left(\frac{1}{100}\right)^2 + \dots$$

Notice this part is a geometric series with $a = \frac{53}{1000}$ and $r = \frac{1}{100}$

$$= 3.1 + \frac{\frac{53}{1000}}{1 - \frac{1}{100}} = \frac{31}{10} + \frac{\frac{53}{1000}}{\frac{99}{100}}$$

$$= \frac{31}{10} + \frac{53}{990} = \frac{3069}{990} + \frac{53}{990} = \frac{3122}{990} \text{ or } \frac{1561}{495}$$