11.4 #6 note: the series has positive terms

by comparison

$5n+4^n > 4^n$

$\frac{1}{5n+4^n} < \frac{1}{4^n}$

so $\frac{3^n-1}{5n+4^n} < \frac{3^n-1}{4^n}$

now $3^n-1 < 3^n$

so $\frac{3^n-1}{4^n} < \frac{3^n}{4^n}$

Together we get

$\frac{3^n-1}{5n+4^n} < \frac{3^n-1}{4^n} < \frac{3^n}{4^n}$

now $\sum \frac{3^n}{4^n} = \sum \left(\frac{3}{4}\right)^n$ is a convergent geometric series $r = 3/4$

by the comparison test.

$\sum \frac{3^n-1}{5n+4^n}$ will converge.

by Limit comparison

Known $\sum \frac{3^n}{4^n} = \sum \left(\frac{3}{4}\right)^n$ is geometric + is convergent $r = 3/4$

$$\lim_{n\to\infty} \frac{\frac{3^n-1}{5n+4^n}}{\frac{3^n}{4^n}} = \lim_{n\to\infty} \frac{3^n-1}{3^n} \cdot \frac{4^n}{5n+4^n} = 1 \cdot 1 = 1$$

by the LCT the Series $\sum \frac{3^n-1}{5n+4^n}$ will converge.

L'Hopital work

$$\lim_{n\to\infty} \frac{3^n-1}{3^n} \overset{L'H}{=} \frac{3^n \cdot \ln(3)}{3^n \ln(3)} = 1$$

$$\lim_{n\to\infty} \frac{4^n}{5n+4^n} \overset{L'H}{=} \lim_{n\to\infty} \frac{4^n \ln(4)}{5+4^n \ln(4)} \overset{L'H}{=} \lim_{n\to\infty} \frac{4^n (\ln(4))^2}{4^n(\ln(4))^2} = 1$$