3) Determine if the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi}{3}\right)}{n^3-5}$$

First thing to check is the test for divergence. Notice that the numerator is bounded between -1 and 1 and as n goes to infinity the denominator will continue to grow. Thus the terms will go to zero as n goes to infinity.

Now notice that the terms of the series are not always positive and they do not alternate signs. This means that the comparison test, the limit comparison test, the integral test, and the alternatign series test will not work.

Try and check for absolute convergence.

Note: $\left|\cos\left(\frac{n\pi}{3}\right)\right| < 1$

Thus. $0 \leq \left|\frac{\cos\left(\frac{n\pi}{3}\right)}{n^3-5}\right| = \frac{\left|\cos\left(\frac{n\pi}{3}\right)\right|}{n^3-5} \leq \frac{1}{n^3-5}$ for $n \geq 2$.

Note: $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges by p-series.

and $\lim_{n\to\infty} \frac{\frac{1}{n^3-5}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{n^3}{n^3-5} = 1$ Thus

$\sum_{n=1}^{\infty} \frac{1}{n^3-5}$ will converge by limit comparison test.

Thus $\sum_{n=1}^{\infty} \left|\frac{\cos\left(\frac{n\pi}{3}\right)}{n^3-5}\right|$ will converge by the comparison test.

Thus $\sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi}{3}\right)}{n^3-5}$ is absolute convergent and therefore convergent.

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