4) The series $\sum a_n$ is defined recursively by

$$a_1 = 1 \qquad a_{n+1} = \frac{(1 + \ln(n))a_n}{n} \text{ for } n \geq 1.$$

Determine if the series converges or diverges.

Since this is a series defines recursively, lets try the ratio test.

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \frac{\frac{1 + \ln(n)}{n} a_n}{a_n} = \lim_{n \to \infty} \frac{1 + \ln(n)}{n}$$

$$\overset{L'H}{=} \lim_{n \to \infty} \frac{\frac{1}{n}}{1} = 0$$

By the ratio test the $\sum a_n$ will be absolute convergent and thus convergent.