Find the radius and the interval of convergence for the power series.

$$\sum_{n=0}^{\infty} \frac{(-3)^n x^{2n}}{n+1}$$

$$\lim_{n\to\infty} \left| \frac{(-3)^{n+1} x^{2n+2}}{n+2} \cdot \frac{n+1}{(-3)^n x^{2n}} \right|$$

$$= \lim_{n\to\infty} \left| -3x^2 \cdot \frac{n+1}{n+2} \right| = |3x^2|$$

To conv. we need

$$|3x^2| < 1$$

$$|x^2| < \frac{1}{3}$$

$$|x| < \sqrt{\frac{1}{3}} \qquad R = \sqrt{\frac{1}{3}}$$

$$-\sqrt{\frac{1}{3}} < x < \sqrt{\frac{1}{3}}$$

now Test the end points

$X = \sqrt{\frac{1}{3}}$

$$\sum \frac{(-3)^n \left(\sqrt{\frac{1}{3}}\right)^{2n}}{n+1} = \sum \frac{(-3)^n \left(\frac{1}{3}\right)^n}{n+1} = \sum \frac{(-1)^n}{n+1}$$

conv. by AST.

$X = -\sqrt{\frac{1}{3}}$

$$\sum \frac{(-3)^n \left(-\sqrt{\frac{1}{3}}\right)^{2n}}{n+1} = \sum \frac{(-3)^n \left(\frac{1}{3}\right)^n}{n+1} = \sum \frac{(-1)^n}{n+1}$$

conv. by AST.

Answer

$R = \sqrt{\frac{1}{3}}$ $\quad$ $I = \left[-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right]$

Mar 31-9:22 PM