3) Find the radius and the interval of convergence for the power series.

$$\sum_{n=0}^{\infty} \frac{3^{2n}(x-2)^n}{n+1}$$

Use the ratio test to find the radius of convergence.

$$\lim_{n\to\infty} \left| \frac{3^{2(n+1)}(x-2)^{n+1}}{n+2} \cdot \frac{n+1}{3^{2n}(x-2)^n} \right|$$

$$= \lim_{n\to\infty} \left| 3^2 (x-2) \cdot \frac{n+1}{n+2} \right| = 9\,|x-2|$$

To have convergence we need this limit to be less than one.

$$9\,|x-2| < 1$$

$$|x-2| < \frac{1}{9} \longrightarrow R = \frac{1}{9}$$

$$-\frac{1}{9} < x-2 < \frac{1}{9}$$

$$\frac{17}{9} < x < \frac{19}{9}$$

$$\sum_{n=0}^{\infty} \frac{3^{2n}(x-2)^n}{n+1}$$

$$\frac{17}{9} < x < \frac{19}{9}$$

now test the endpoints of the interval.

$X = \frac{19}{9}$

$$\sum_{n=0} \frac{3^{2n}\left(\frac{19}{9}-2\right)^n}{n+1} = \sum_{n=0} \frac{3^{2n}\left(\frac{1}{9}\right)^n}{n+1}$$

Note $3^{2n} = (3^2)^n = 9^n$

$$\sum_{n=0} \frac{1}{n+1}$$

This series will diverge by the limit comparison test with the harmonic series(p-series with p=1).

$X = \frac{17}{9}$

$$\sum_{n=0} \frac{3^{2n}\left(\frac{17}{9}-2\right)^n}{n+1} = \sum_{n=0} \frac{9^n\left(-\frac{1}{9}\right)^n}{n+1}$$

$$= \sum \frac{(-1)^n}{n+1}$$

This series will converge by the Alternating series test(AST).

Conclusion:

$R = \frac{1}{9}$

$I = \left[\frac{17}{9}, \frac{19}{9}\right)$