Example: Find the radius and the interval of convergence for the power series.

$$\sum_{n=0}^{\infty} \frac{(2x+1)^{2n}}{8^n}$$

$$a_{n+1} = \frac{(2x+1)^{2(n+1)}}{8^{n+1}} = \frac{(2x+1)^{2n+2}}{8^{n+1}} \qquad a_n = \frac{(2x+1)^{2n}}{8^n}$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \underbrace{\frac{(2x+1)^{2n+2}}{8^{n+1}}}_{a_{n+1}} \cdot \underbrace{\frac{8^n}{(2x+1)^{2n}}}_{\frac{1}{a_n}} \right|$$

$$= \lim_{n\to\infty} \left|\frac{(2x+1)^2}{8}\right| = \left|\frac{(2x+1)^2}{8}\right| < 1$$

$$|(2x+1)^2| < 8$$

$$(2x+1)^2 < 8$$

$$\sqrt{(2x+1)^2} < \sqrt{8} = 2\sqrt{2}$$

$$|2x+1| < \sqrt{8} \longrightarrow |2(x+\tfrac{1}{2})| < \sqrt{8} = 2\sqrt{2}$$

$$|x+\tfrac{1}{2}| < \sqrt{2}$$

$$R = \sqrt{2}$$

$$-\sqrt{8} < 2x+1 < \sqrt{8}$$

$$-\sqrt{8}-1 < 2x < \sqrt{8}-1$$

$$\frac{-\sqrt{8}-1}{2} < x < \frac{\sqrt{8}-1}{2}$$

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now test endpoint

$$\sum \frac{(2x+1)^{2n}}{8^n}$$

$(\sqrt{8})^{2n} = \left((\sqrt{8})^2\right)^n$

$= 8^n$

$x = \frac{\sqrt{8}-1}{2}$

$$\sum \frac{\left(2\left(\frac{\sqrt{8}-1}{2}\right)+1\right)^{2n}}{8^n} = \sum \frac{(\sqrt{8}-1+1)^{2n}}{8^n}$$

$$= \sum \frac{(\sqrt{8})^{2n}}{8^n} = \sum \frac{8^n}{8^n} = \sum 1$$

d.V.

$x = \frac{-\sqrt{8}-1}{2}$

$$\sum \frac{\left(2\left(\frac{-\sqrt{8}-1}{2}\right)+1\right)^{2n}}{8^n} = \sum \frac{(-\sqrt{8}-1+1)^{2n}}{8^n}$$

$$= \sum \frac{(-\sqrt{8})^{2n}}{8^n} = \sum \frac{8^n}{8^n} = \sum_{n=0}^{\infty} 1$$

d.V.

$R = \sqrt{2}$

$I: \left(-\frac{\sqrt{8}-1}{2}, \frac{\sqrt{8}-1}{2}\right)$