Chapter 1. **Introduction to vectors and vector functions**

Section 1.1 **Vectors**

Definition. A **two-dimensional vector** is an ordered pair $\vec{a} =< a_1, a_2 >$ of real numbers. The numbers a_1 and a_2 are called the **components** of $\vec{a}$.

A **representation** of the vector $\vec{a} =< a_1, a_2 >$ is a directed line segment $\vec{AB}$ from any point $A(x, y)$ to the point $B(x + a_1, y + a_2)$.

A particular representation of $\vec{a} =< a_1, a_2 >$ is the directed line segment $\vec{OP}$ from the origin to the point $P(a_1, a_2)$, and $\vec{a} =< a_1, a_2 >$ is called the **position vector** of the point $P(a_1, a_2)$.

Given the points $A(x_1, y_1)$ and $B(x_2, y_2)$, then $\boxed{\vec{AB} =< x_2 - x_1, y_2 - y_1 >}$.

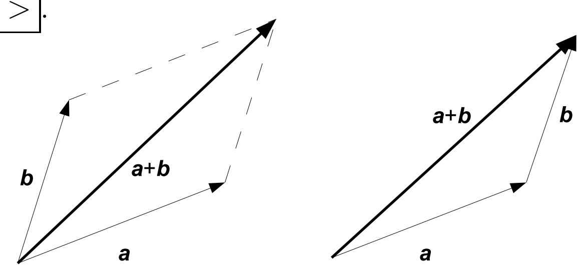
The **magnitude (length)** $|\vec{a}|$ of $\vec{a}$ is the length of any its representation.

The length of $\vec{a}$ is $\boxed{|\vec{a}| = \sqrt{a_1^2 + a_2^2}}$

The length of the vector $\vec{AB}$ from $A(x_1, y_1)$ to $B(x_2, y_2)$ is $\boxed{|\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$

The only vector with length 0 is the **zero vector** $\vec{0} =< 0, 0 >$. This vector is the only vector with no specific direction.

Vector addition If $\vec{a} =< a_1, a_2 >$ and $\vec{b} =< b_1, b_2 >$, then the vector $\vec{a} + \vec{b}$ is defined by $\boxed{\vec{a} + \vec{b} =< a_1 + b_1, a_2 + b_2 >}$.



Parallelogram Law *Triangular Law*

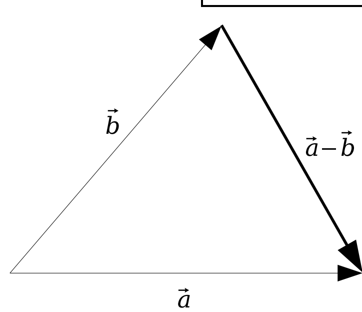
Multiplication of a vector by a scalar If c is a scalar and $\vec{a} =< a_1, a_2 >$, then the vector is defined by $\boxed{c\vec{a} =< ca_1, ca_2 >}$.

$\boxed{|c\vec{a}| = c|\vec{a}|}$

Two vectors $\vec{a}$ and $\vec{b}$ are called **parallel** if $\vec{b} = c\vec{a}$ for some scalar c. If $\vec{a} =< a_1, a_2 >$, $\vec{b} =< b_1, b_2 >$, then $\vec{a}$ and $\vec{b}$ are parallel if and only if $\boxed{\frac{b_1}{a_1} = \frac{b_2}{a_2}}$.

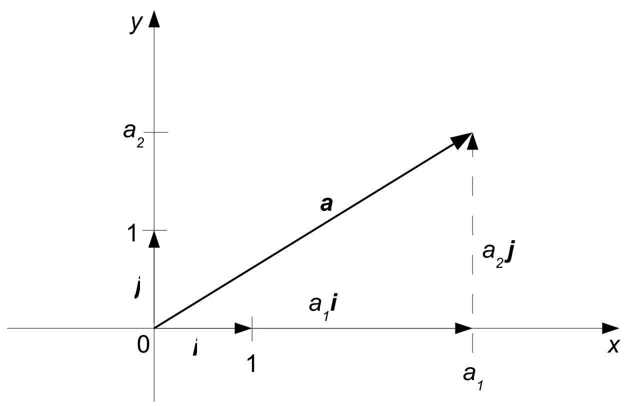
By the **difference** of two vectors, we mean $\boxed{\vec{a} - \vec{b} = \vec{a} + (-\vec{b})}$

so, if $\vec{a} =< a_1, a_2 >$ and $\vec{b} =< b_1, b_2 >$, then $\boxed{\vec{a} - \vec{b} =< a_1 - b_1, a_2 - b_2 >}$.

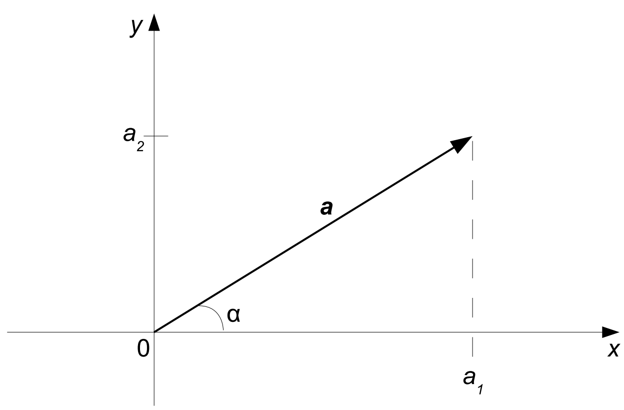


Properties of vectors If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are vectors and k and m are scalars, then
1. $\vec{a}+\vec{b}=\vec{b}+\vec{a}$
2. $\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$
3. $\vec{a}+\vec{0}=\vec{a}$
4. $\vec{a}+(-\vec{a})=\vec{0}$
5. $k(\vec{a}+\vec{b})=k\vec{a}+k\vec{b}$
6. $(k+m)\vec{a}=k\vec{a}+m\vec{a}$
7. $(km)\vec{a}=k(m\vec{a})$
8. $1\vec{a}=\vec{a}$

Let $\vec{\imath} =< 1,0 >$ and $\vec{\jmath} =< 0,1 >$, $|\vec{\imath}|=|\vec{\jmath}|=1$. $\boxed{\vec{a} =< a_1, a_2 >= a_1\vec{\imath}+a_2\vec{\jmath}}$



$\mathbf{a}=<a_1,a_2>=a_1\mathbf{i}+a_2\mathbf{j}$



$\mathbf{a}=<a_1,a_2>=|\mathbf{a}|<\cos\alpha,\sin\alpha>$

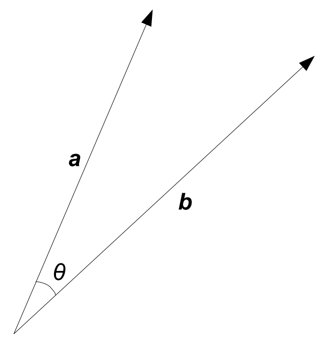
$\alpha=\cos^{-1}\left\{\frac{a_1}{|\vec{a}|}\right\}=\sin^{-1}\left\{\frac{a_2}{|\vec{a}|}\right\}$

A **unit vector** is a vector whose length is 1.

A vector $\boxed{\vec{u}=\frac{1}{|\vec{a}|}\vec{a}=\left\langle\frac{a_1}{|\vec{a}|},\frac{a_2}{|\vec{a}|}\right\rangle}$ is a unit vector that has the same direction as $\vec{a} =< a_1, a_2 >$.

Section 1.2 **The dot product**

Definition. The **dot** or **scalar product** of two nonzero vectors $\vec{a}$ and $\vec{b}$ is the number $\boxed{\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta}$ where θ is the angle between $\vec{a}$ and $\vec{b}$, $0 \le \theta \le \pi$. If either $\vec{a}$ or $\vec{b}$ is $\vec{0}$, we define $\vec{a}\cdot\vec{b} = 0$.



$\vec{a}\cdot\vec{b} > 0$ if and only if $0 < \theta < \pi/2$
$\vec{a}\cdot\vec{b} < 0$ if and only if $\pi/2 < \theta < \pi$

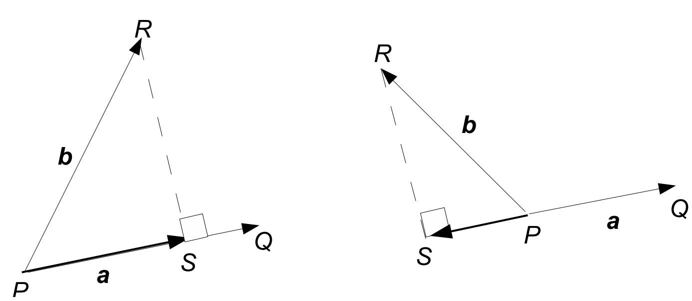
Two nonzero vectors $\vec{a}$ and $\vec{b}$ are called **perpendicular** or **orthogonal** if the angle between them is $\pi/2$.

Two vectors $\vec{a}$ and $\vec{b}$ are orthogonal if and only if $\vec{a}\cdot\vec{b} = 0$.

If $\vec{a} = < a_1, a_2 >$ and $\vec{b} = < b_1, b_2 >$, then $\boxed{\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2}$.

$$\boxed{\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}}$$

Properties of the dot product If $\vec{a}$, $\vec{b}$, an $\vec{c}$ are vectors and k is a scalar, then
1. $\vec{a}\cdot\vec{a} = |a|^2$
2. $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$
3. $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$
4. $(k\vec{a})\cdot\vec{b} = k(\vec{a}\cdot\vec{b}) = \vec{a}(k\vec{b})$
5. $\vec{0}\cdot\vec{a} = 0$

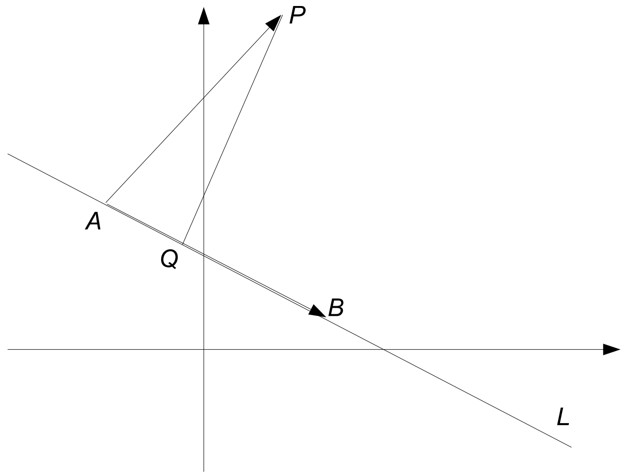


$\vec{PS}$ =$\text{proj}_{\vec{a}}\vec{b}$ is called the **vector projection of $\vec{b}$ onto $\vec{a}$**.
$|\vec{PS}|$ =$\text{comp}_{\vec{a}}\vec{b}$ is called the **scalar projection of $\vec{b}$ onto $\vec{a}$** or the **component of $\vec{b}$ along** $\vec{a}$.

$$\boxed{\text{comp}_{\vec{a}}\vec{b} = \left|\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right|} \quad \boxed{\text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2} < a_1, a_2 >}$$

Definition Given the nonzero vector $\vec{a} = < a_1, a_2 >$, the **orthogonal complement** of $\vec{a}$ is the vector $\boxed{\vec{a}^{\perp} = < -a_2, a_1 >}$.

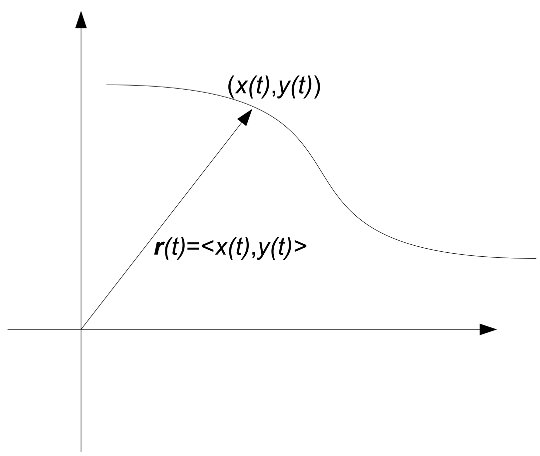
Vectors $\vec{a}$ and $\vec{a}^{\perp}$ are orthogonal and $|\vec{a}| = |\vec{a}^{\perp}|$



The distance from the point P to the line L $\boxed{PQ = \text{comp}_{\vec{AB}^{\perp}} \vec{AP}}$.

Section 1.3 **Vector functions**

Definition The curve of a type $x = x(t)$, $y = y(t)$ is called a **parametric curve** and the variable t is called a **parameter**.

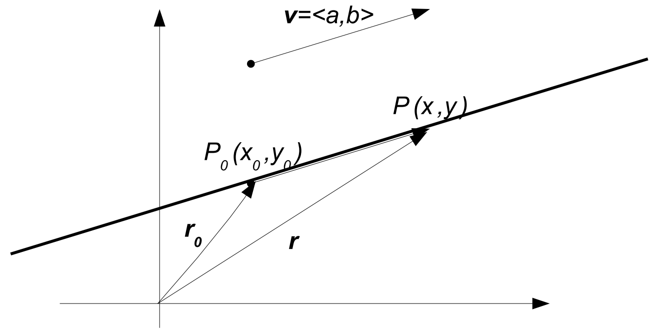


Definition Vector $\vec{r}(t) =< x(t), y(t) >= x(t)\vec{\imath} + y(t)\vec{\jmath}$ is called the **position vector** for the point with coordinates $(x(t), y(t))$.

A function such as $\vec{r}(t)$, whose range is a set of vectors, is called a **vector function** of t.

Sometimes we can convert the parametric equation of a curve to an equation involving only x and y; this form is called a *Cartesian equation*.

A line L is determined by a point $P_0(x_0, y_0)$ on L and a direction. Let $\vec{v} =< a, b >$ be a vector parallel to line L. Let $P(x, y)$ be be an arbitrary point on L and let $\vec{r_0} =< x_0, y_0 >$ and $\vec{r} =< x(t), y(t) >$ be the position vectors of P_0 and P. Then the *vector equation* of line L is



$$\boxed{\vec{r}(t) = \vec{r_0} + t\vec{v} =< x_0 + at, y_0 + bt >}$$

If $\vec{r} =< x(t), y(t) >$, $\vec{v} =< a, b >$ and $P(x_0, y_0)$ then *parametric equations* of the line L are

$$\boxed{x(t) = x_0 + at,\ y(t) = y_0 + bt.}$$