

1. Find the area of the region bounded by  $y = x^2 + 1$ ,  $y = 3 - x^2$ ,  $x = 0$ , and  $x = 2$ .
2. Find the volume of the solid obtained by rotating the region bounded by  $y = x^2 - 1$ ,  $y = 0$ ,  $x = 1$ ,  $x = 2$  about the  $x$ -axis.
3. Find the volume of the solid obtained by rotating the region bounded by  $y = x^2$ ,  $y = 0$ ,  $x = 1$ ,  $x = 2$  about
  - (a) the  $y$ -axis
  - (b)  $x = 4$
4. A heavy rope, 50 ft long, weighs 0.5 lb/ft and hangs over the edge of a building 120 ft high. How much work is done in pulling the rope to the top of the building?
5. A spring has a natural length of 20 cm. If a 25-N force is required to keep it stretched to a length of 30 cm, how much work is required to stretch it from 20 cm to 25 cm?
6. Find the average value of  $f = \sin^2 x \cos x$  on  $[-\pi/2, \pi/4]$ .

7. Evaluate the integral

(a)  $\int t^2 \cos(1 - t^3) dt$

(b)  $\int \frac{x^2}{\sqrt{1-x}} dx$

(c)  $\int_0^1 x^2 e^{-x} dx$

(d)  $\int \sin^3 x \cos^4 x dx$

(e)  $\int_0^{\pi/8} \sin^2(2x) \cos^3(2x) dx$

(f)  $\int \sin^2 x \cos^4 x dx$

(g)  $\int_0^{\pi/4} \tan^4 x \sec^2 x dx$

(h)  $\int \tan x \sec^3 x dx$

(i)  $\int \sin 3x \cos x dx$

(j)  $\int \frac{x^2}{\sqrt{5-x^2}} dx$

(k)  $\int \frac{x^3}{\sqrt{x^2+4}} dx$

(l)  $\int \frac{dx}{\sqrt{x^2+4x-5}}$

$$(m) \int \frac{dx}{x^2(x^2 + 1)}$$

$$(n) \int \frac{x^2 + 3x - 1}{x - 1} dx$$

$$(o) \int_0^{\infty} \frac{dx}{(x + 2)(x + 3)}$$

$$(p) \int_{-\infty}^1 \frac{dx}{(2x - 3)^2}$$

$$(q) \int_4^5 \frac{dx}{(5 - x)^{2/5}}$$

8. Find the length of the curve  $x(t) = 3t - t^3$ ,  $y(t) = 3t^2$ ,  $0 \leq t \leq 2$ .
9. Find the area of the surface obtained by rotating the curve  $y = x^3$ ,  $0 \leq x \leq 2$  about the  $x$ -axis.
10. Find the area of the surface obtained by rotating the curve  $x = \sqrt{2y - y^2}$ ,  $0 \leq y \leq 1$  about the  $y$ -axis.
11. (a) Sketch the curve  $r = 2(1 + \cos \theta)$  in polar coordinates.  
(b) Find the length of the polar curve  $r = 2(1 + \cos \theta)$ .
12. A region  $D$  lies inside the circle  $r = 3 \sin \theta$  and outside the cardioid  $r = 1 + \sin \theta$ .
- (a) Sketch the region  $D$ .  
(b) Find the area of the region  $D$ .

13. Find the following limits

$$(a) \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\ln n}$$

$$(b) \lim_{n \rightarrow \infty} \frac{1 - 2n^2}{\sqrt[3]{n^6 + 1} + 2n^2}$$

$$(c) \lim_{n \rightarrow \infty} (\sqrt{n + 1} - \sqrt{n})$$

14. Find the sum of the series

$$(a) \sum_{n=1}^{\infty} \frac{2^{2n+1}}{3^{3n-1}}$$

$$(b) \sum_{n=2}^{\infty} \frac{(-1)^n x^2}{n!}$$

$$(c) \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n} (2n)!}$$

15. Which of the following series is convergent?

$$(a) \sum_{n=1}^{\infty} \frac{n^2}{n^{5/7} + 1}$$

$$(b) \sum_{n=1}^{\infty} \frac{\cos^2 n}{3^n}$$

$$(c) \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

16. Which of the following series is absolutely convergent?

$$(a) \sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$$

$$(b) \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$$

$$(c) \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{\sqrt{n-2}}$$

$$(d) \sum_{n=0}^{\infty} (-1)^n \frac{2^{2n}}{3^{3n}}$$

17. Find the radius of convergence and interval of convergence of the series  $\sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$ .

18. Find the power series representation for the function  $f(x) = \ln(3-2x)$  centered at 0.

19. Find the Taylor series for  $f(x) = xe^x$  at  $x = 2$ .

20. Find the Maclaurin series for  $f(x) = x \sin(x^3)$ .