

MATH 308. Differential Equations

Homework 1

Nataliya Goncharuk, natasha_goncharuk@tamu.edu

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1. (3 pt) Solve the differential equation

$$y'(x) = x \sin(x^2) - \frac{1}{1+x^2}$$

with initial condition $y(0) = 0$.

2. The boat is pushed away from the berth. Let $x(t)$ be the position of the boat at time t . Let $v(t)$ be its velocity at time t .

Suppose that the velocity satisfies the differential equation

$$m \frac{dv}{dt} = -kv,$$

where $m > 0$ is the mass of the boat and $k > 0$ is the drag coefficient. Suppose that the initial position is $x(0) = 0$ and the initial velocity is $v(0) = 1$.

- a) (1 pt) Find $v(t)$.
- b) (2 pt) Find $x(t)$.
- c) (1 pt) Find the limit position of the boat $\lim_{t \rightarrow \infty} x(t)$. Does it increase or decrease with m ?
3. (3 pt) Plot the direction field for the equation $y'(x) = 1 - y^2$ (manually or using a computer). Sketch several solution curves. Mark the solution curve that corresponds to $y(0) = 0.5$. Using a picture, find $\lim_{x \rightarrow +\infty} y(x)$ for the solution with initial condition $y(0) = 0.5$.