MATH 308. Differential Equations

Homework 10

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Task 1. (4 pt) Using the eigenvalue method, solve the systems with initial conditions $x_1(t) = 1, x_2(t) = 0$:

(a) $x_1' = x_1 + 3x_2, x_2' = -2x_2$;

(b) $x_1' = 5x_1 + x_2, x_2' = -3x_1 + x_2$.

Task 2. (2+1 pt) (a) Use the eigenvalue method to solve the system of three equations $x' = Ax$ with the matrix

$$A = \begin{pmatrix} -4 & 1 & 1 \\ 0 & -3 & 1 \\ 1 & 2 & -3 \end{pmatrix}$$

You may use a computer algebra system[1]to compute eigenvalues and eigenvectors.

(b) Show that components of any solution (e.g. the functions $x_1(t), x_2(t), x_3(t)$) all tend to zero as $t \to +\infty$.

Task 3. (3 pt) Solve the following initial value problems **without** the eigenvalue method, using your knowledge in first- and second-order linear equations.

(a) $x_1' = x_2, x_2' = -9x_1$, initial conditions $x_1(0) = 0, x_2(0) = 3$;

[1]e.g. go to `live.sympy.org`, define A = Matrix([[a, b,c], [d, e, f], [g, h, k]]) and execute A.eigenvals(), A.eigenvects().

(b) $x_1' = x_2, x_2' = -10x_1 - 2x_2$, same initial conditions;
(c) $x_1' = 2x_1 + x_2, x_2' = 2x_2$, same initial conditions.

In each case, plot the corresponding phase curve using a graphing calculator[2]. In which cases $x_1(t)$ will change sign more than once as $t \to +\infty$?

Hint: in (a), (b), differentiate the first equation to get a second-order differential equation on x_1 (you will have to get rid of x_2). In (c), solve the second equation first.

[2]e.g. `desmos.com` can plot parametric curves if you use t as a parameter