MATH 308. Differential Equations

Homework 4

Nataliya Goncharuk, natasha_goncharuk@tamu.edu

Deadline: Sep 22, 11:00 pm

Task 1. (2+2+1 pt) The position of the damped oscillator is governed by the following equation:

$$my'' = -ky - cy'.$$

Set $m = 1, k = 5$.

(a) Solve the equation for $c = 4$ (underdamped oscillator) with the initial condition $y(0) = 0, y'(0) = 1$.

(b) Solve the equation for $c = 6$ (overdamped oscillator) with the initial condition $y(0) = 0, y'(0) = 1$.

(c) Plot the graphs of your solution $y(t)$ from (a) and (b) (manually or using a computer). In which case the system will oscillate around the equilibrium $y(0) = 0$?

Task 2. (2 pt) For the equation $x^2y'' - xy' + y = 0$, use the sympy function dsolve() to find the general solution[1]. Also find the fundamental set of solutions. Include the command and its output in your solution.

Task 3. (3 pt) Consider the equation

$$x^2y'' + 2xy' - 2y = 0.$$

[1] You can visit https://live.sympy.org/ to run the python code online: e.g. the command dsolve(diff(f(t), t, 2) - diff(f(t), t) - f(t)) means "solve $\frac{d^2f}{dt^2} - \frac{df}{dt} - f(t) = 0$"

It is easy to see that $y_1(x) = x$ is a solution of this equation. Use the following steps to find the general solution.

(a) Suppose that $y_2(x) = x \cdot f(x)$ is also a solution. Show that the Wronskian $W(x) = y_1 y_2' - y_1' y_2$ of these solutions equals $x^2 \cdot f'(x)$.

(b) Use the Abel's formula to find the Wronskian. Now find $f(x)$ and thus $y_2(x) = x \cdot f(x)$.

Comment: it is sufficient to find one solution y_2 here — you do not need to find all possible y_2.

(c) Write out the general solution of this equation.

Comment: the statement of this problem has changed: I replaced $p(x)$ with $f(x)$ to avoid the duplication in notation, since we often denote coefficients of the second-order equation by p, q. If your solution uses old notation $y_2 = x \cdot p(x)$, it is no need to update your solution — but make sure this does not lead to mistakes.