MATH 308. Differential Equations

Homework 5

Nataliya Goncharuk, natasha_goncharuk@tamu.edu

Deadline: Sep 29, 11:00 pm

Task 1 (1+1+2+2 pt)**.** Solve the following nonhomogeneous ODEs.

(a) $y'' - 4y' + 3y = e^x$;

(b) $y'' = y + e^x \cos(2x)$;

(c) $y'' + 2y' + y = 8\cosh x$;

Comment: cosh is a hyperbolic cosine,

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

(d) $y'' + y = \frac{1}{\sin t}$.

Task 2 (2+1+1 pt)**.** The periodic outer force $F = \sin \omega t$ is applied to a damped oscillator. The motion off the oscillator is described by the equation

$$y'' = -3y - cy' + \sin \omega t,$$

$c > 0$.

(a) Find a periodic solution of this equation in the form $y(t) = a\cos \omega t + b \sin \omega t$ (the answer will depend on c, ω). Prove that the amplitude of this solution is $((3 - \omega^2)^2 + c^2\omega^2)^{-0.5}$.

(b) Fix c (you may assume that c is small) and find $\omega = \omega_{max} > 0$ such that the amplitude of this periodic solution reaches its maximum at ω. You will find the "resonant" frequency of the outer force for the damped oscillator.

Comment: you will see that for larger c, the amplitude monotonically decreases as ω increases, and thus there is no maximum point $\omega_{max} > 0$.

Hint: you may wish to denote $\nu = \omega^2$ when searching for the maximum, but remember that $\nu \geq 0$.

(c) For $c = 2$, write out the general solution of this equation and show that all solutions tend to the periodic solution as $t \to +\infty$.