MATH 308. Differential Equations

Homework 6

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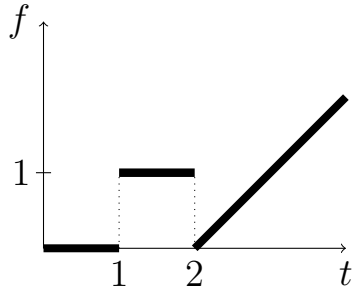
Deadline: 11:00 pm

Task 1. (3 pt) The function $f(t)$ is given by the following graph. Write out the formula for f and compute its Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ using the definition of the Laplace transform.



Task 2. (3 pt) Using the table of Laplace transforms, find inverse Laplace transforms of the following functions:

$$(a)\ \frac{1}{s^2-2s+1}; \quad (b)\ \frac{s+1}{s^2+2s+2}; \quad (c)\ \frac{1}{s^4+4s^2}.$$

Hint: Find these functions in the table on p.252. For (c), represent as a sum of functions from the table using partial fractions.

Task 3. (4 pt) Suppose that the function y is a solution of the differential equation $y'' - 2y' + y = h(t)$ with initial conditions $y(0) = 0, y'(0) = 0$.

(a) Show that its Laplace transform $Y = \mathcal{L}\{y\}$ satisfies

$$Y(s) = \frac{\mathcal{L}\{h(t)\}}{s^2 - 2s + 1}.$$

Hint: apply the Laplace transform to both sides of the differential equation, then use the formula for $\mathcal{L}\{f'\}$.

(b) Use this formula to find the solution of the equation

$$y'' - 2y' + y = t^{10}e^t, \qquad y(0) = y'(0) = 0.$$