MATH 308. Differential Equations

Homework 8

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Deadline: Oct 27, 11:00 pm

In this home assignment, you may use the Table of Laplace transforms (p.252 of the textbook).

Task 1. (3 pt)

(a) Compute

$$\mathcal{L}\{\int_0^t (t-\tau)^2 \sin\tau d\tau\}.$$

You are not required to compute the integral itself.

(b) Compute $e^t * e^{2t}$ and verify that $\mathcal{L}\{e^t\} \cdot \mathcal{L}\{e^{2t}\}$ is indeed equal to $\mathcal{L}\{e^t * e^{2t}\}$.

(c) Express the solution of the initial value problem

$$y'' + 2y' + 2y = g(t), \quad y(0) = 1, y'(0) = 2$$

in terms of a convolution integral.

Task 2. (3 pt) Solve the initial value problem

$$y'' + 3y' + 2y = \delta(t-3) - \delta(t-4), \quad y(0) = 0, y'(0) = 4$$

and plot the graph of the solution.

To check your answer: the resulting function $y(t)$ will satisfy the differential equation, will be continuous, and its derivative will be discontinuous at $t = 3, 4$ (you can check this visually after you plot the graph of $y(t)$).

Task 3. (4 pt)

The mass-spring system experiences 3 impulses of the same magnitude at time $a, 2a$, and $3a$:

$$y'' + y = \delta(t-a) + \delta(t-2a) + \delta(t-3a); \qquad y(0) = 0, y'(0) = 1.$$

(a) Treating a as a parameter, solve the equation. Observe that after the sequence of impulses ends, the motion of the system becomes periodic.

(b) Show that after the sequence of impulses ends, the amplitude of oscillation of the system is at most 4. Find at least one value of a such that this amplitude is equal to 4. Plot the graph of the solution $y(t)$ in the domain $t > 0$ for this value of a.