MATH 308. Differential Equations

Homework 9

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Deadline: Nov 3, 11:00 pm

Task 1. (4 pt) (a) Find the first 5 coefficients a_0, a_1, a_2, a_3, a_4 of the power series expansion at zero $y(x) = \sum_{n=0}^{\infty} a_n x^n$ for the solution of the initial value problem

$$(1-x)y'' = y, \quad y(0) = 1, y'(0) = 2.$$

(b) Use the Fuchs' theorem (Theorem 5.3.1 in the textbook) to estimate the radius of convergence for this power series.

Task 2. (3 pt) (a) Find the power series solution of the equation

$$y'' - xy' - y = 1, \quad y(0) = 1, y'(0) = 0.$$

(b) Determine whether 0 is a local maximum/minimum of the solution $y(x)$.

Hint: If the Taylor series of some function starts with $a_0 + a_n x^n + \dots$ and $a_n \neq 0$, then this function behaves roughly as $a_0 + a_n x^n$ near zero. In particular, it will have a minimum/maximum at zero if and only if $a_0 + a_n x^n$ has a minimum/maximum at zero.

Task 3. (3 pt) The function $y(x) = \sum_{n=0}^{\infty} a_n x^n$ is a solution of the equation

$$xy'' + y' + xy = 0, y(0) = 1, y'(0) = 0.$$

(a) Find the power series for $y(x)$.

(b) Plot the graph of the Taylor's polynomial $a_0 + a_1 x + \cdots + a_8 x^8$ that approximates $y(x)$. Compare it to the graph of $y(x)$ that appears below. Can we use the approximation $y(x) \approx a_0 + a_1 x + \cdots + a_8 x^8$ for $x = 2$? For $x = 5$?