MATH 308. Differential Equations

Lecture 1. Introduction

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August 25, 2022

Differential equations

ODE

$$\frac{dy}{dx} = F(x, y(x))$$

Initial condition $y(x_0) = y_0$;

Solution of the initial value problem : a function $y(x)$ that satisfies both the equation and the initial value condition.

Example

Which of the following functions is a solution of the differential equation $y'(x) = 1$ with initial condition $y(0) = 1$?

(A) $y(x) = x^2$;

(B) $y(x) = x$;

(C) $y(x) = x + 1$;

(D) $y(x) = 1$.

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Differential Equations Around Us

Second Newton's law:

$$m\frac{dv}{dt} = F(t, x, v)$$

$m\frac{dv}{dt} = -mg$	Free fall
$m\frac{dv}{dt} = -av$	Drag force, low speed, no gravity
$m\frac{dv}{dt} = -av^2$	Drag force, high speed, no gravity
$m\frac{dv}{dt} = -mg - av$	Free fall in the air (water drop)
$m\frac{dv}{dt} = -mg - av^2$	Free fall in the air (skydiver)

Equations from populational biology:

$\frac{dy}{dx} = ay$	Population growth
$\frac{dy}{dx} = ay - c$	Population growth (with harvesting)

Verifying whether a function is a solution-1

Which of the following functions is a solution of the differential equation $\frac{dv}{dt} = -at$? Can you guess more solutions of this equation?

(A) $v(t) = -\frac{at^2}{2}$;

(B) $v(t) = -\frac{av^2}{2}$;

(C) $v(t) = ae^t$;

(D) $v(t) = 5e^{-at}$.

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- Simplify: $y(x) = \int f(x)\,dx$.

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- Let us solve an ODE of the form $y'(x) = f(x)$.
- Integrate both sides: $\int y'(x)\,dx = \int f(x)\,dx$.
- Simplify: $y(x) = \int f(x)\,dx$.
- More precisely: $y(x) = y(x_0) + \int_{x_0}^{x} f(\xi)\,d\xi$.

Verifying whether a function is a solution-2

Which of the following functions is a solution of the differential equation $\frac{dv}{dt} = -av$? Can you guess more solutions of this equation?

(A) $v(t) = -\frac{at^2}{2}$;

(B) $v(t) = -\frac{av^2}{2}$;

(C) $v(t) = ae^t$;

(D) $v(t) = 5e^{-at}$.

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Direction fields

(discussion on the whiteboard)

Summary

Solution curve Graph of solution;

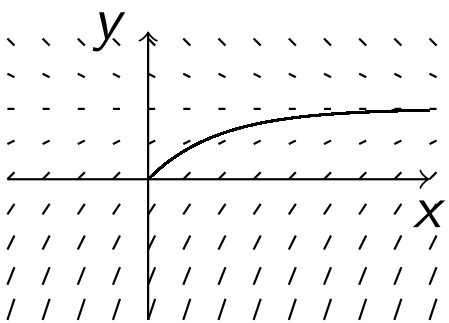
Direction field Direction with slope $F(x, y)$ at each point (x, y).

Solution curves are tangent to all directions of the direction field

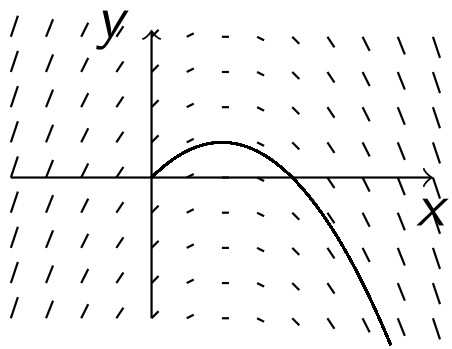
Plotting direction fields

The students were asked to plot the direction field of the equation $y' = 1 - y$ and the solution curve that corresponds to the solution with initial condition $y(0) = 0$. Which plot is correct?

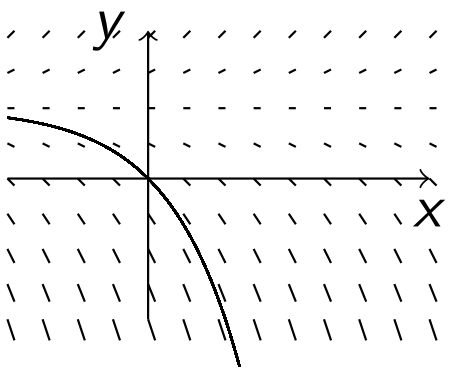
(a)



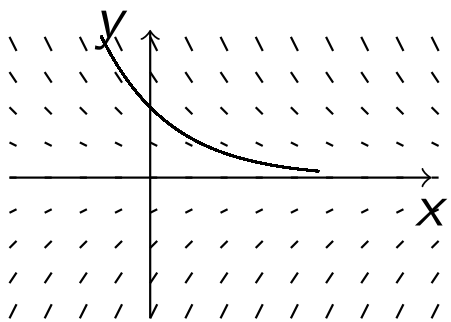
(b)



(c)



(d)



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